

# A Brief Introduction to Causal Inference

Brady Neal

[causalcourse.com](https://causalcourse.com)

# What is causal inference?

Inferring the effects of any treatment/policy/intervention/etc.

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- Effect of social media on mental health

# What is causal inference?

Inferring the effects of any treatment/policy/intervention/etc.

Examples:

- Effect of treatment on a disease
- Effect of climate change policy on emissions
- Effect of social media on mental health
- Many more (effect of  $X$  on  $Y$ )

**Motivating example: Simpson's paradox**

**Correlation does not imply causation**

**Then, what does imply causation?**

**Causation in observational studies**

## **Motivating example: Simpson's paradox**

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# Simpson's paradox: COVID-27

New disease: COVID-27



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Treatment T: A (0) and B (1)

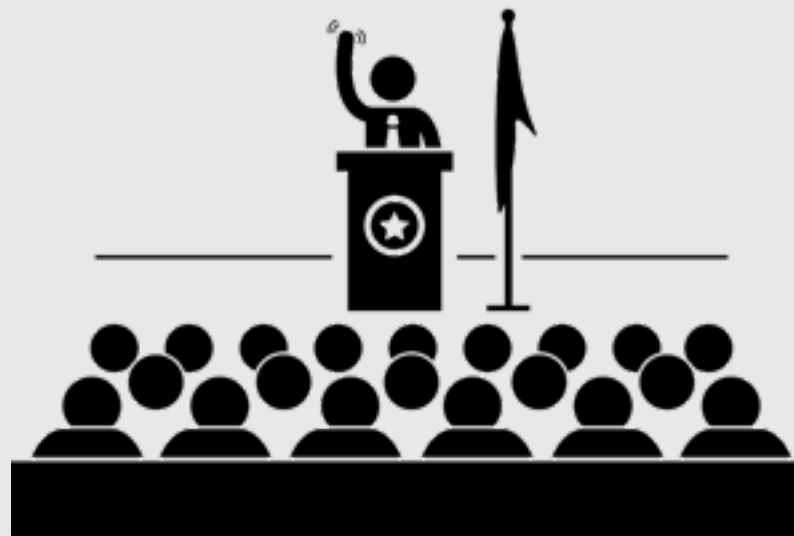
# Simpson's paradox: COVID-27

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Treatment T: A (0) and B (1)

YOU



# Simpson's paradox: COVID-27

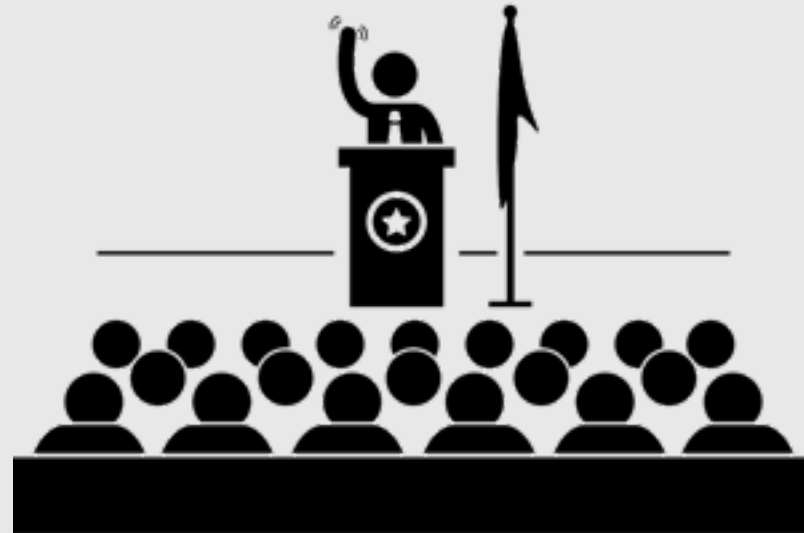
New disease: COVID-27



Treatment T: A (0) and B (1)

Condition C: mild (0) or severe (1)

**YOU**



# Simpson's paradox: COVID-27

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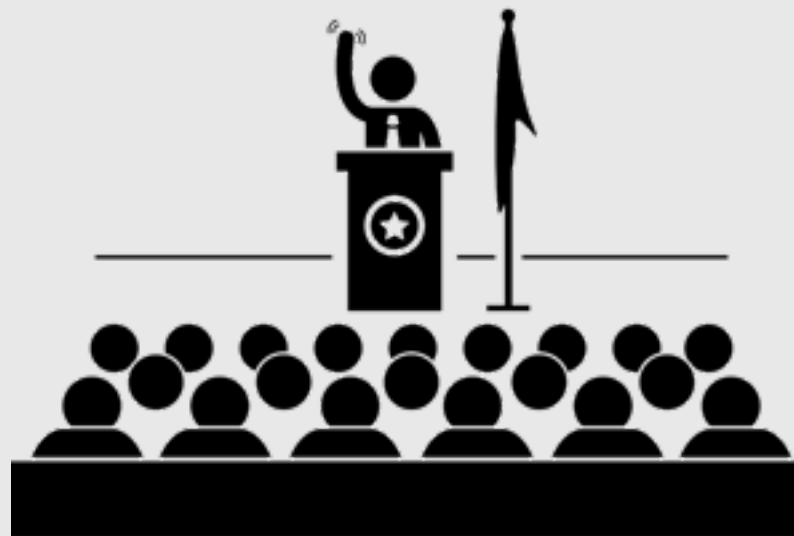


Treatment T: A (0) and B (1)

Condition C: mild (0) or severe (1)

Outcome Y: alive (0) or dead (1)

**YOU**



# Simpson's paradox: mortality rate table

	Total
Treatment	
A	<b>16%</b> (240/1500)
B	19% (105/550)

$\mathbb{E}[Y|T]$

# Simpson's paradox: mortality rate table

		Condition		
		Mild	Severe	Total
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)
	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)

$\mathbb{E}[Y|T, C = 0]$      $\mathbb{E}[Y|T, C = 1]$      $\mathbb{E}[Y|T]$

# Simpson's paradox: mortality rate table

		Condition			
		Mild	Severe	Total	
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)	$\frac{1400}{1500} (0.15) + \frac{100}{1500} (0.30) = 0.16$
	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)	$\frac{50}{550} (0.10) + \frac{500}{550} (0.20) = 0.19$
		$\mathbb{E}[Y T, C = 0]$	$\mathbb{E}[Y T, C = 1]$	$\mathbb{E}[Y T]$	



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	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)	$\frac{50}{550} (0.10) + \frac{500}{550} (0.20) = 0.19$
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<b>Treatment</b>	A	15% (210/ <u>1400</u> )	30% (30/ <u>100</u> )	<b>16%</b> (240/1500)	$\frac{1400}{1500}(0.15) + \frac{100}{1500}(0.30) = 0.16$
	B	<b>10%</b> (5/ <u>50</u> )	<b>20%</b> (100/ <u>500</u> )	19% (105/550)	$\frac{50}{550}(0.10) + \frac{500}{550}(0.20) = 0.19$
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<b>Treatment</b>	A	15% (210/ <u>1400</u> )	30% (30/ <u>100</u> )	<b>16%</b> (240/1500)	$\frac{1400}{1500}(0.15)$	$+$	$\frac{100}{1500}(0.30)$	$= 0.16$
	B	<b>10%</b> (5/ <u>50</u> )	<b>20%</b> (100/ <u>500</u> )	19% (105/550)	$\frac{50}{550}(0.10)$	$+$	$\frac{500}{550}(0.20)$	$= 0.19$
		$\mathbb{E}[Y T, C = 0]$	$\mathbb{E}[Y T, C = 1]$	$\mathbb{E}[Y T]$				

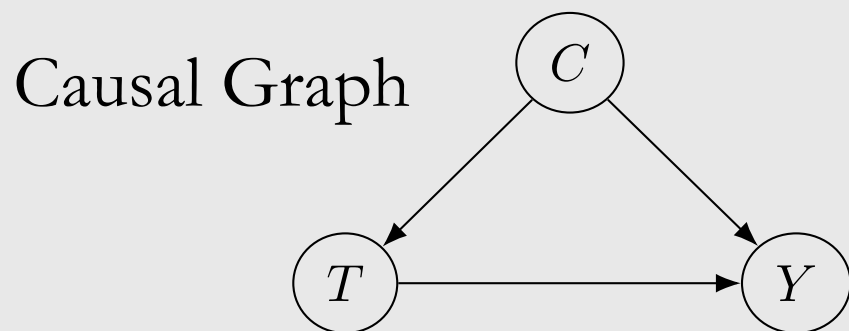
Which treatment should you choose?

# Simpson's paradox: scenario 1 (treatment B)

		Condition		
		Mild	Severe	Total
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)
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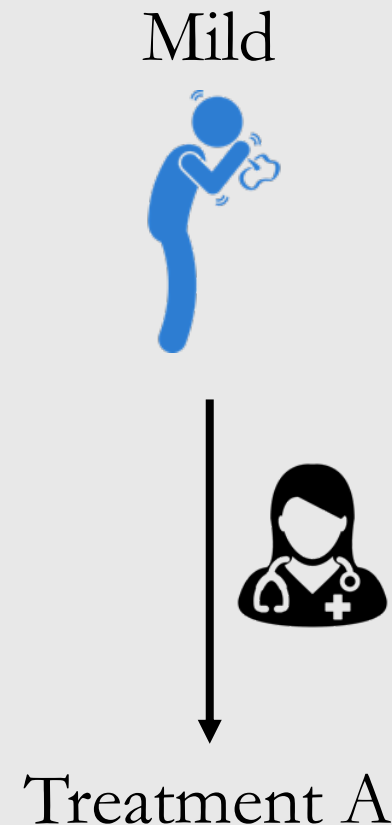
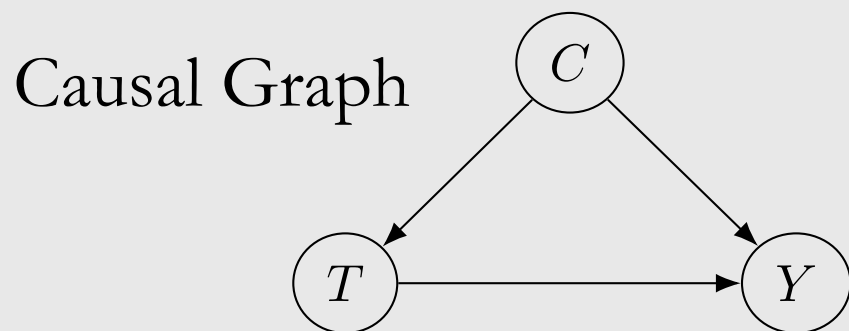
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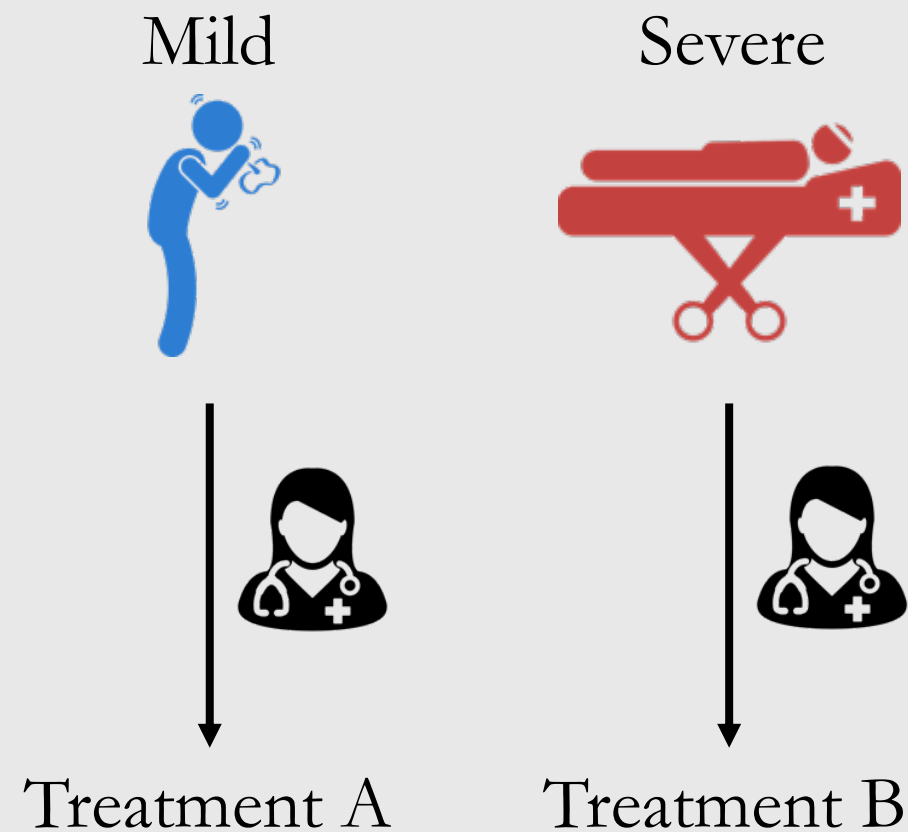
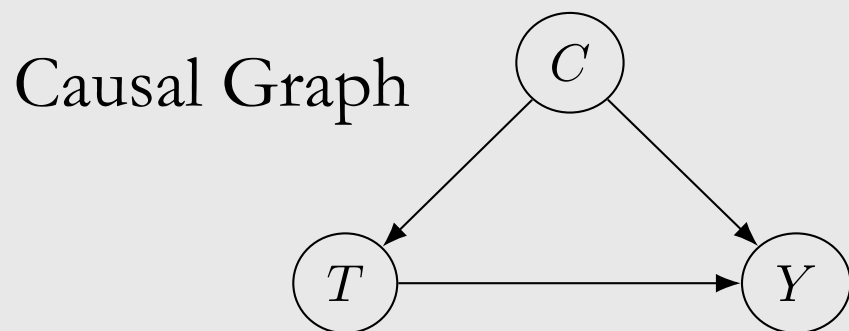
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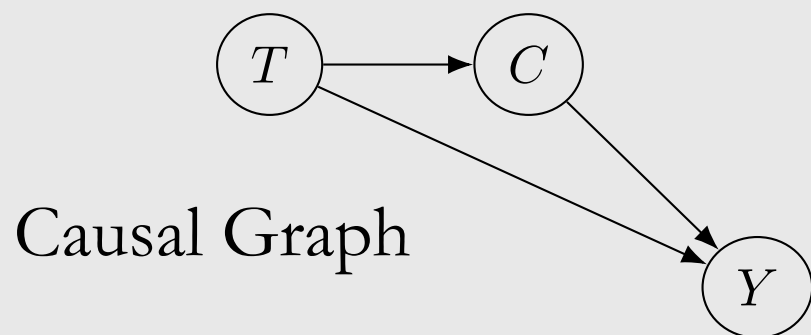
# Simpson's paradox: scenario 2 (treatment A)

		Condition		
		Mild	Severe	Total
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)
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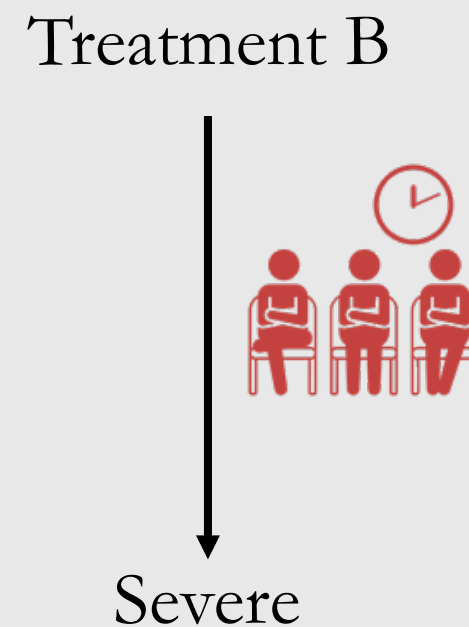
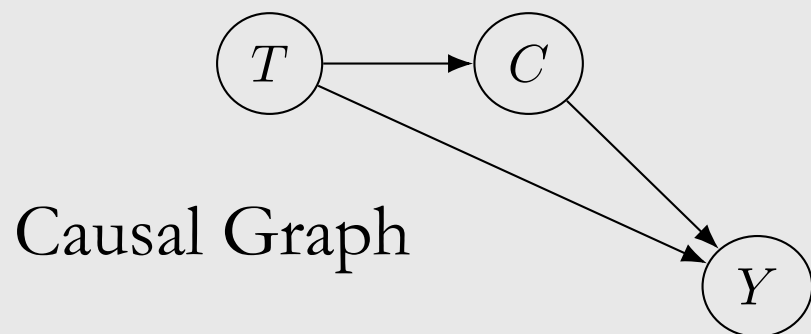
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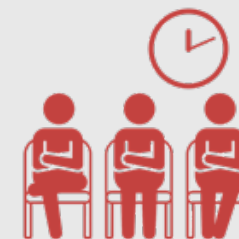
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Treatment A

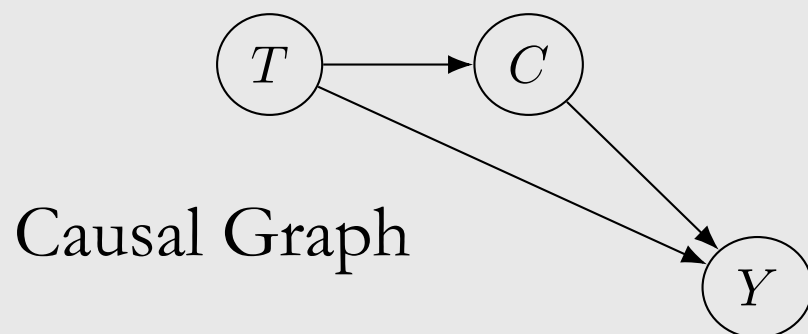


Mild

Treatment B



Severe

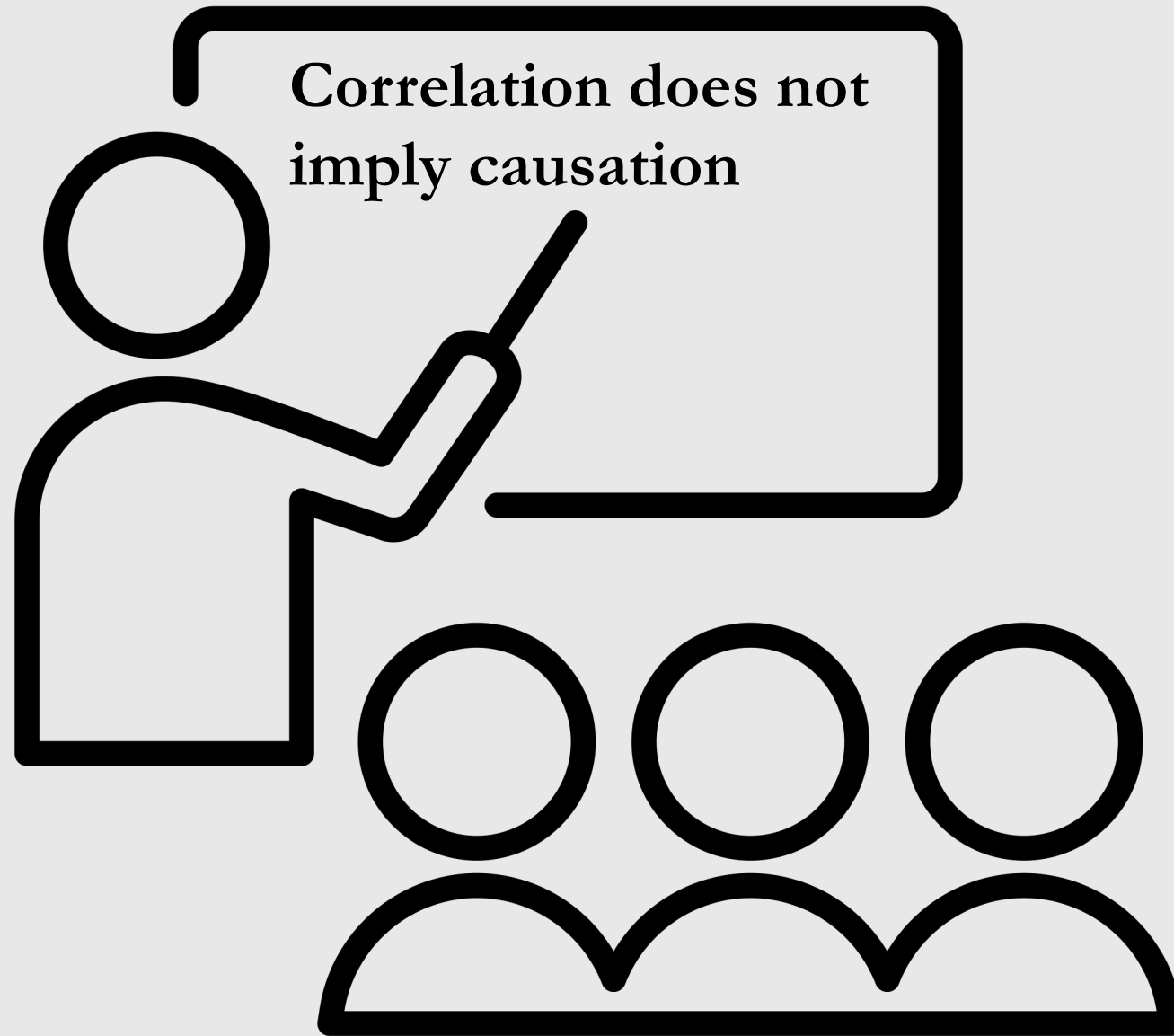


Motivating example: Simpson's paradox

**Correlation does not imply causation**

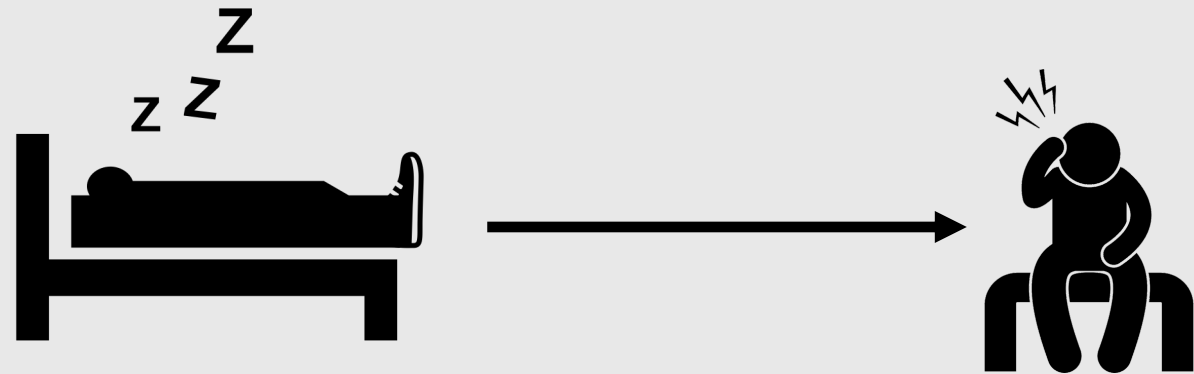
Then, what does imply causation?

Causation in observational studies



# Correlation does not imply causation

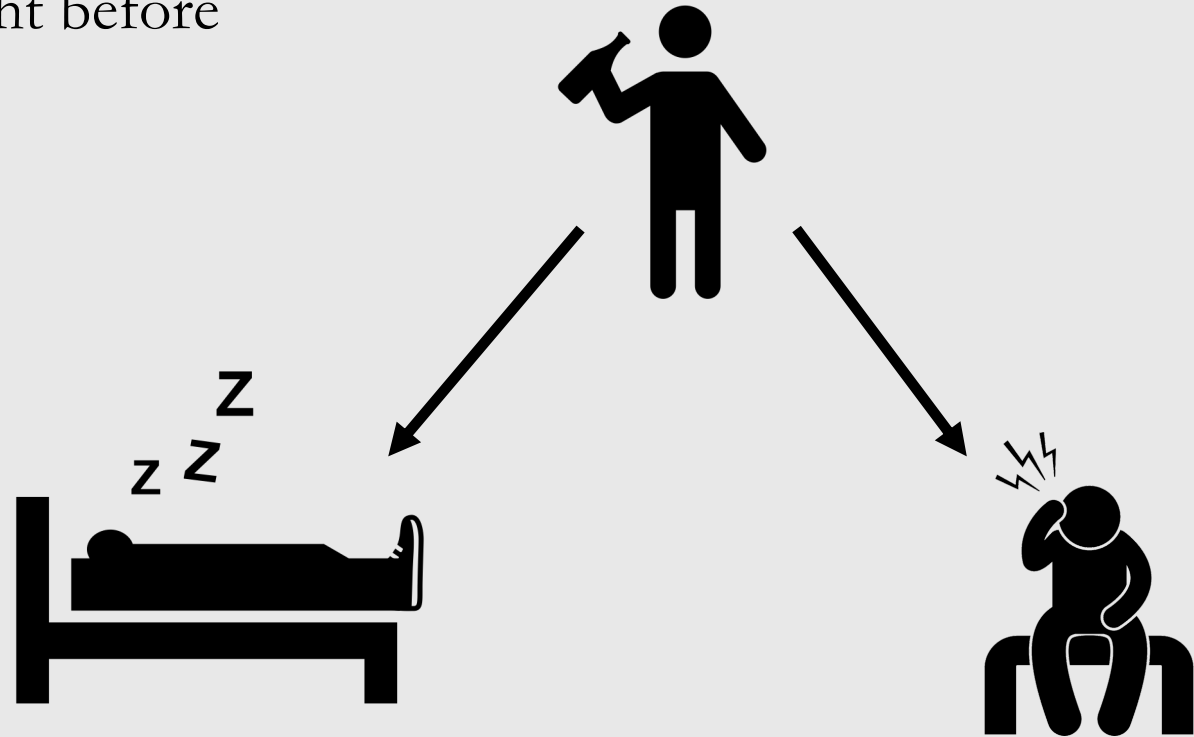
Sleeping with shoes on is strongly correlated with waking up with a headache



# Correlation does not imply causation

Sleeping with shoes on is strongly correlated with waking up with a headache

Common cause: drinking the night before

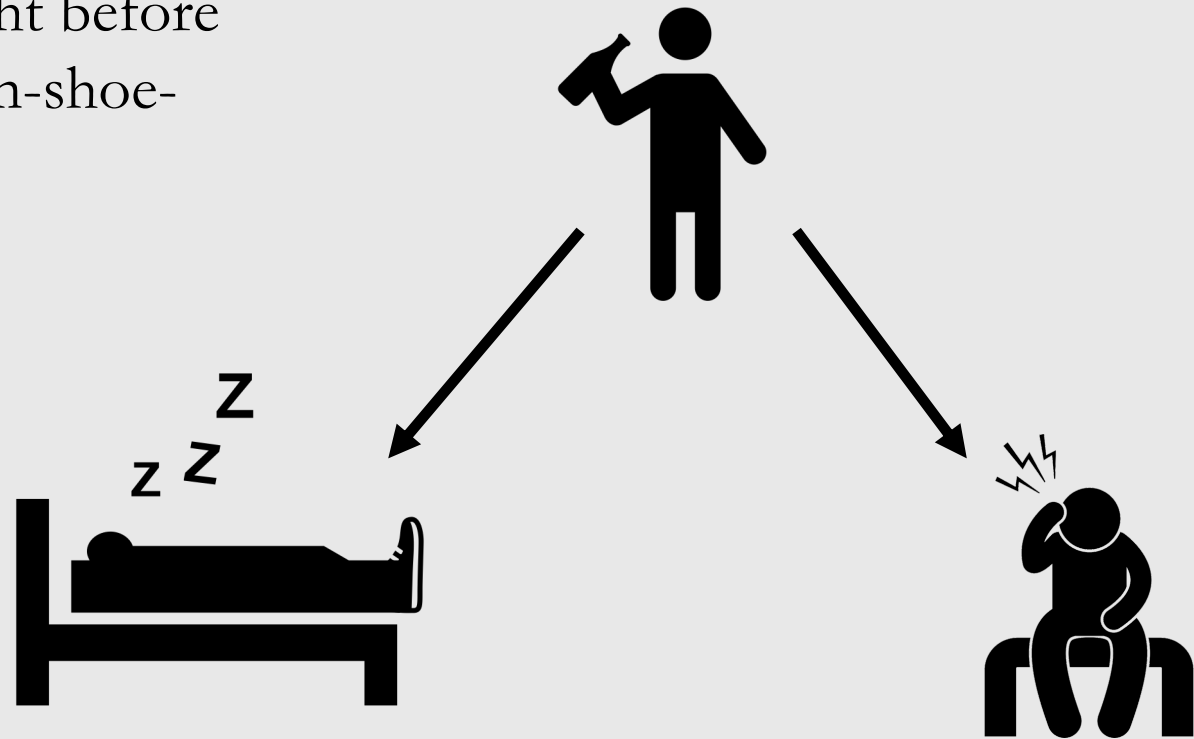


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Common cause: drinking the night before

1. Shoe-sleepers differ from non-shoe-sleepers in a key way



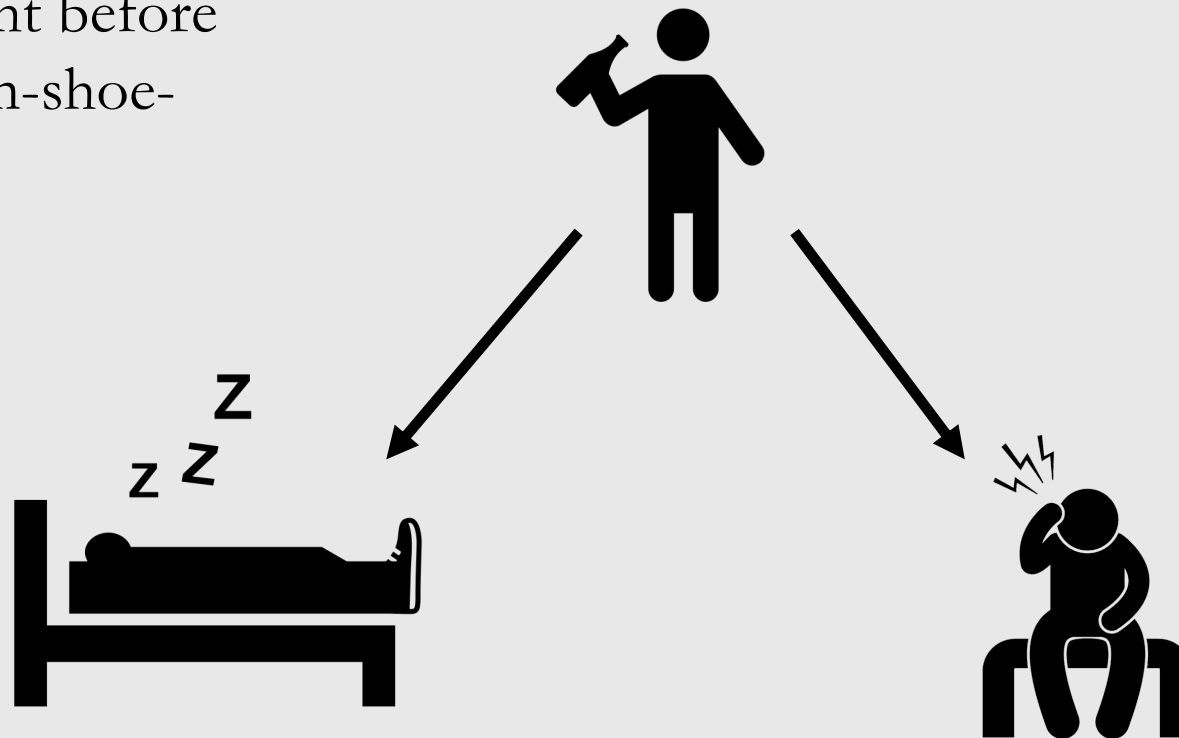


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Sleeping with shoes on is strongly correlated with waking up with a headache

Common cause: drinking the night before

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2. Confounding

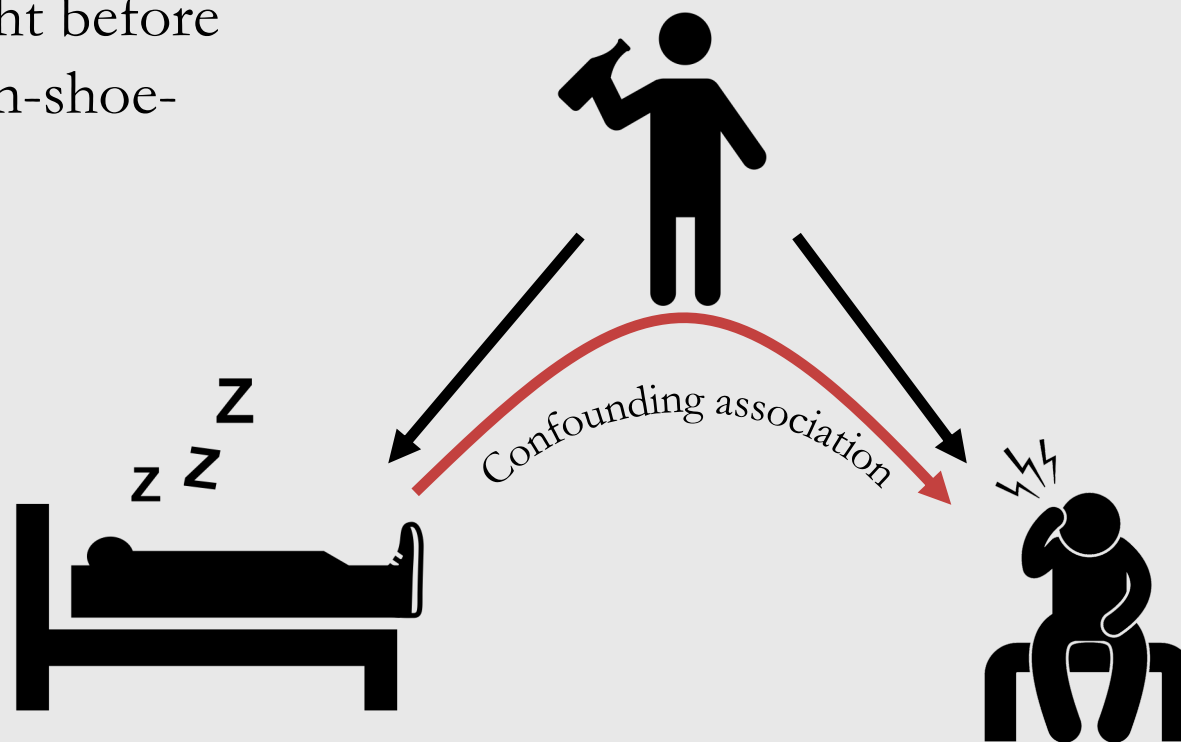


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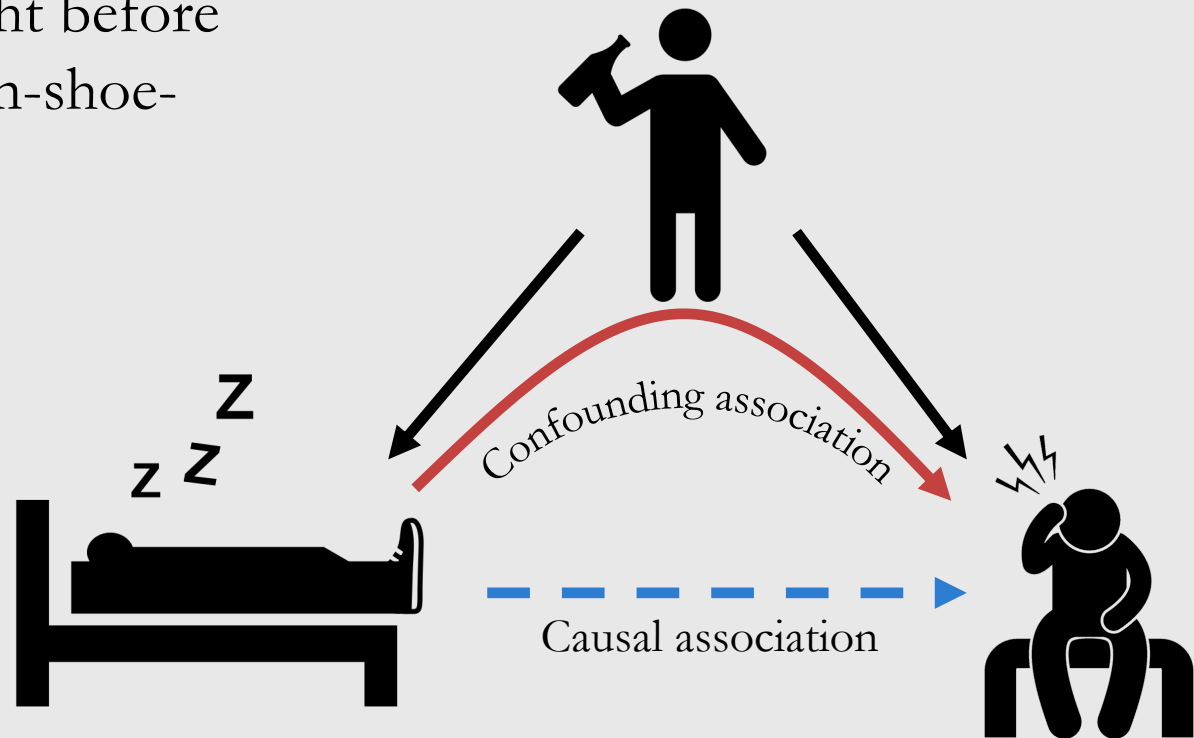


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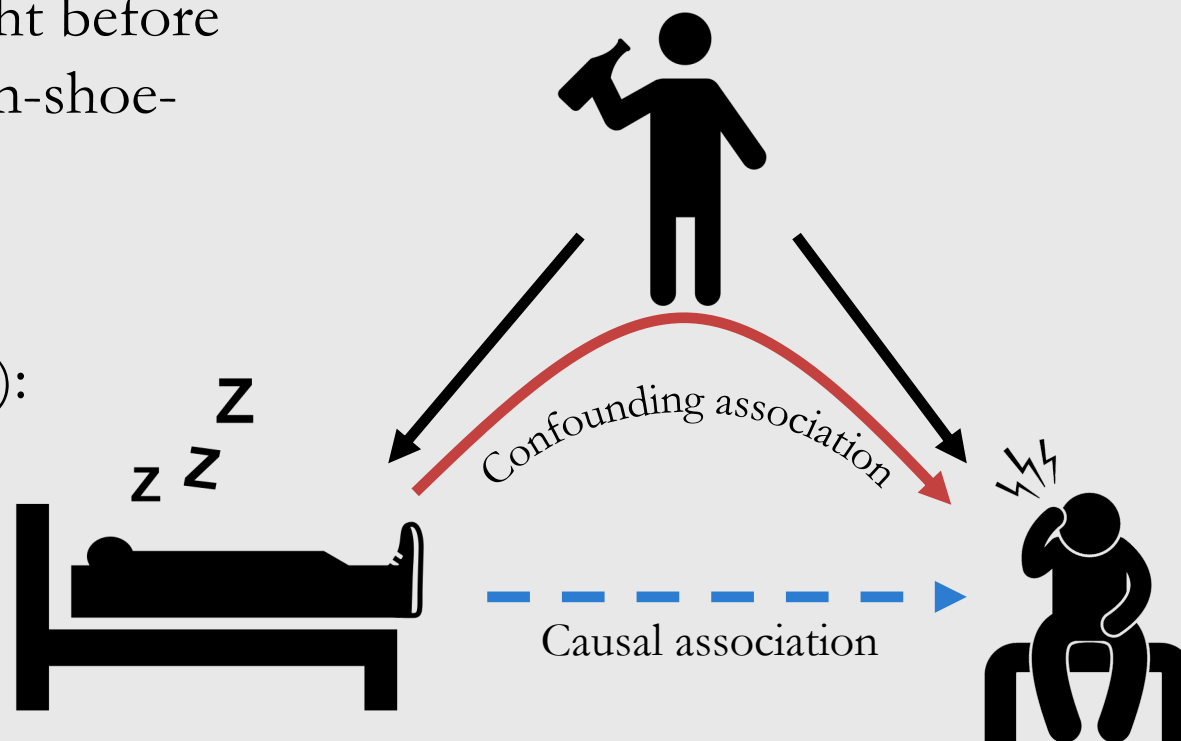
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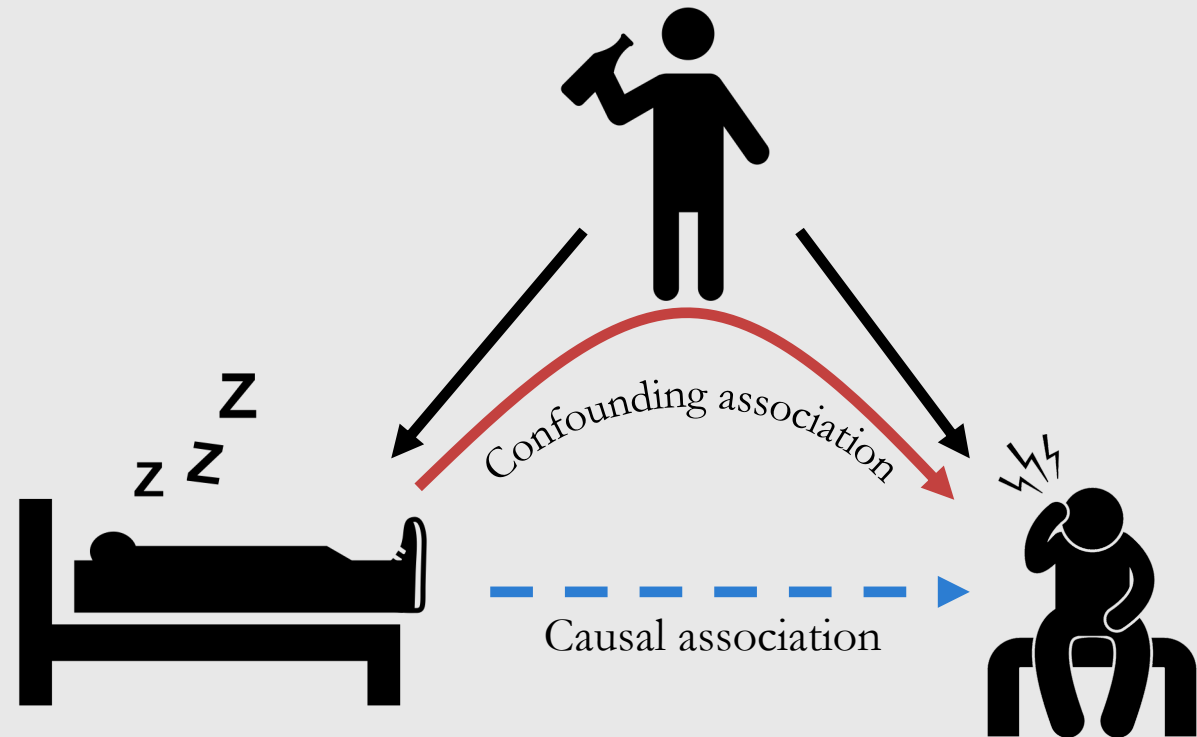
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Total association (e.g. correlation):  
mixture of causal and  
confounding association

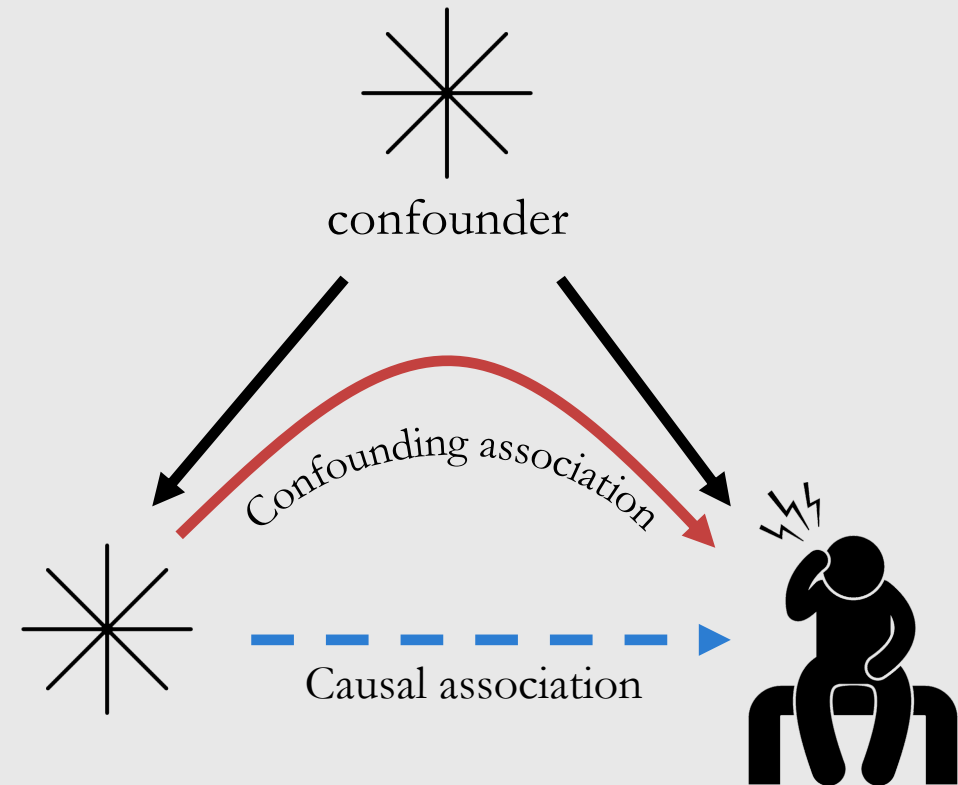


# “Correlation = Causation” is a cognitive bias<sup>1</sup>



<sup>1</sup>[The illusion of causality: A cognitive bias underlying pseudoscience \(Blanco & Matute, 2018\)](#)

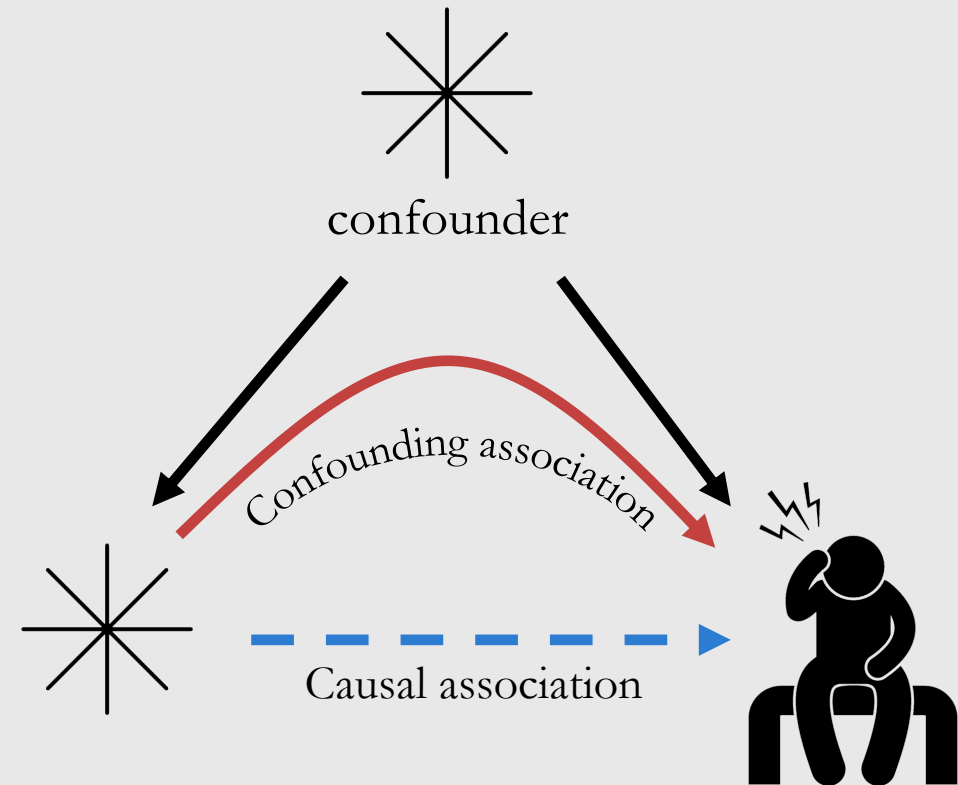
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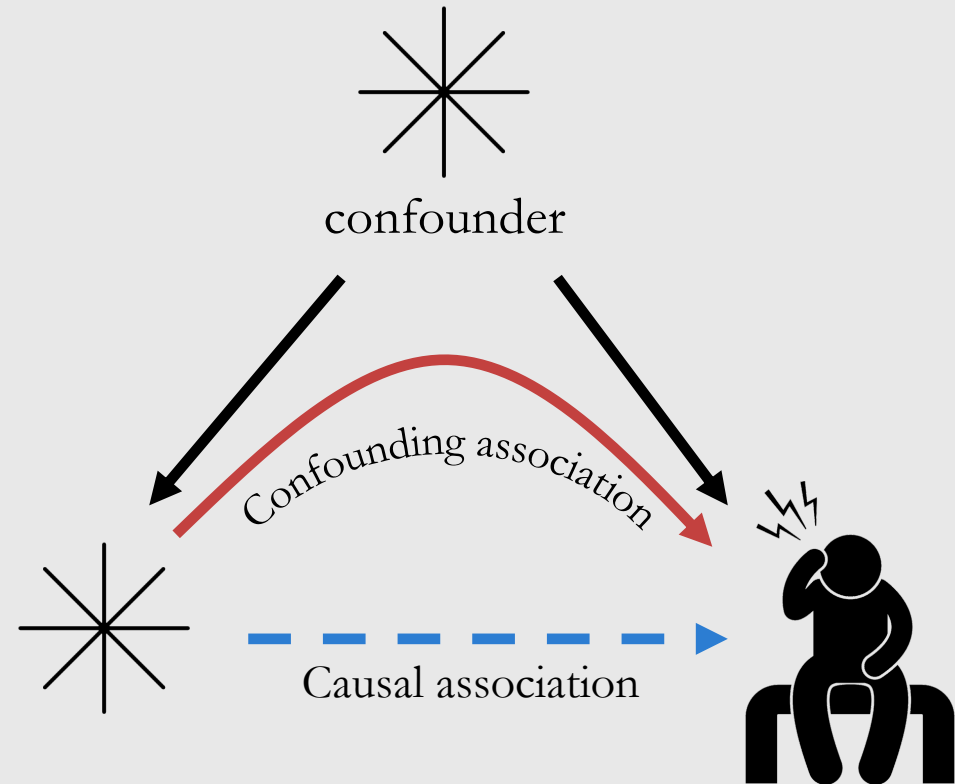
Availability heuristic (another cognitive bias) gives us ✱



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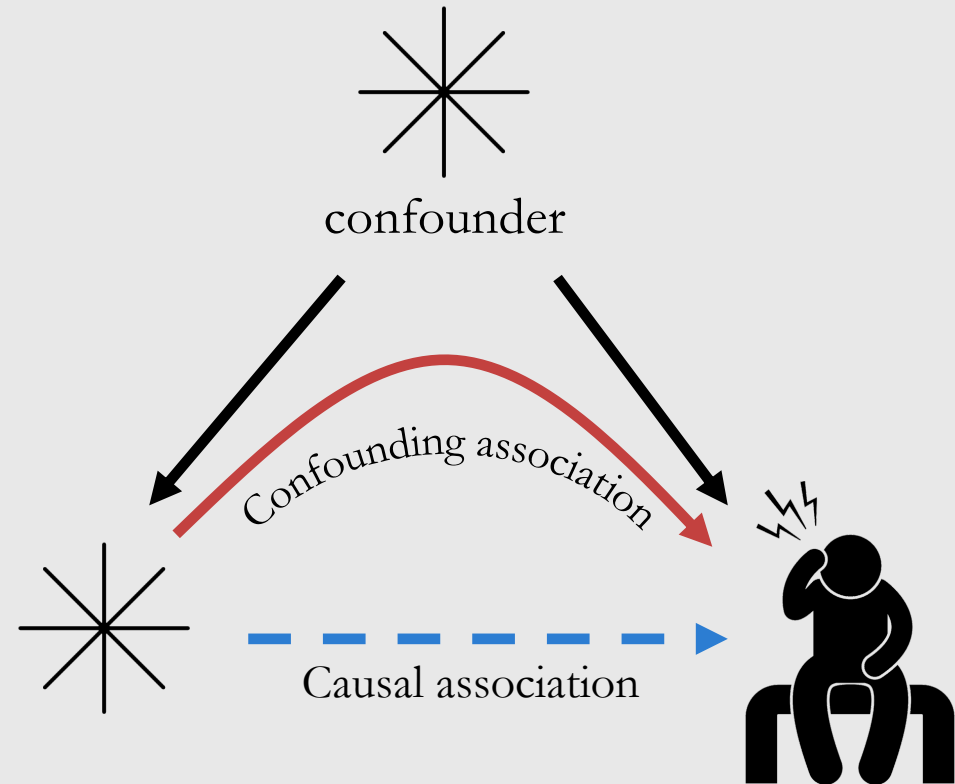
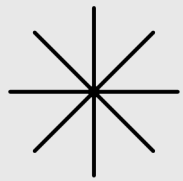


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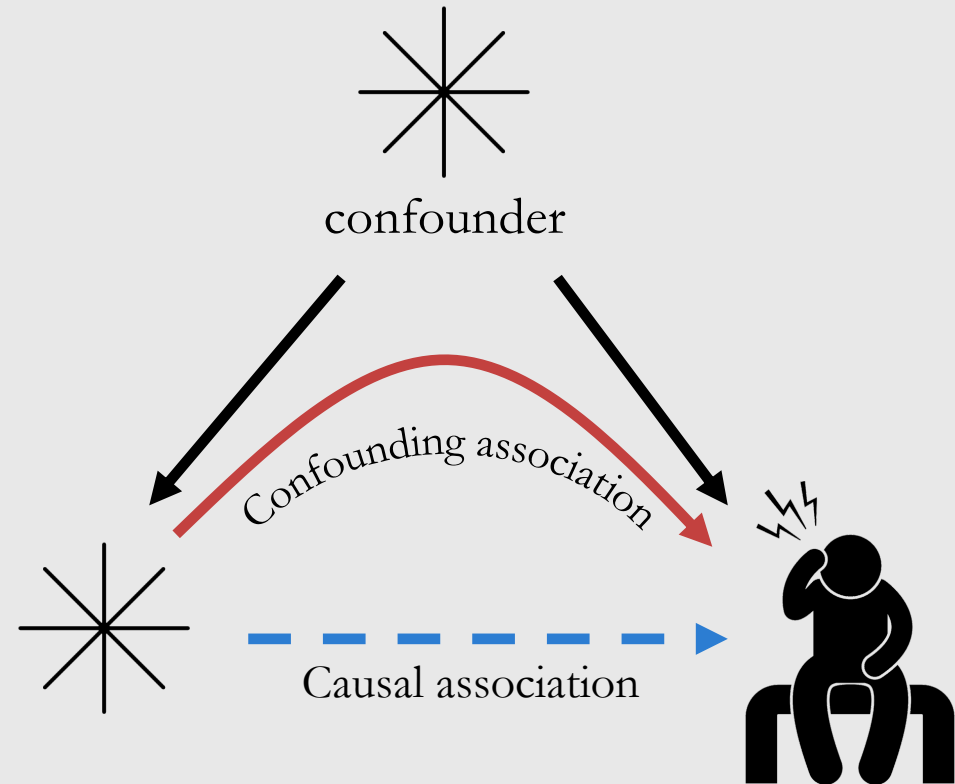
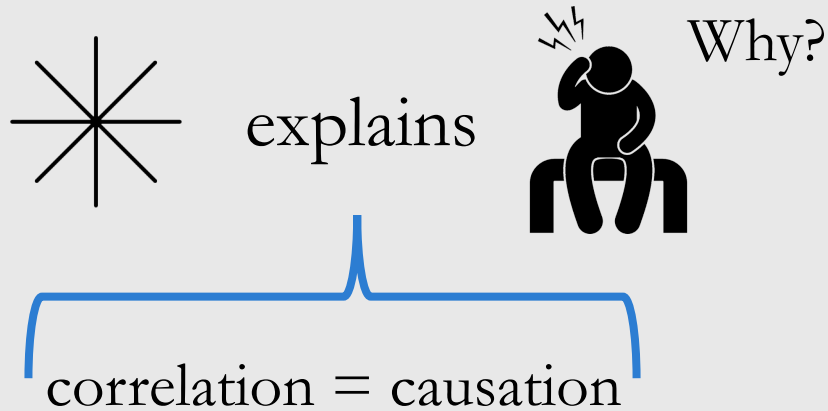
Availability heuristic (another cognitive bias) gives us ✱  
Motivated reasoning (another cognitive bias)



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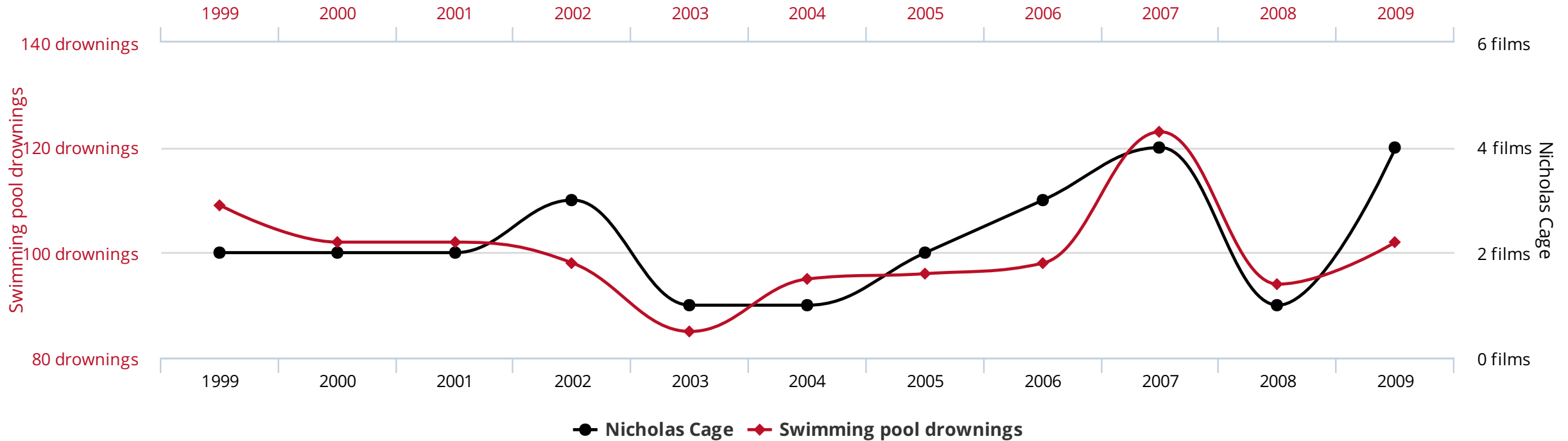
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# Nicolas Cage drives people to drown themselves

**Number of people who drowned by falling into a pool**

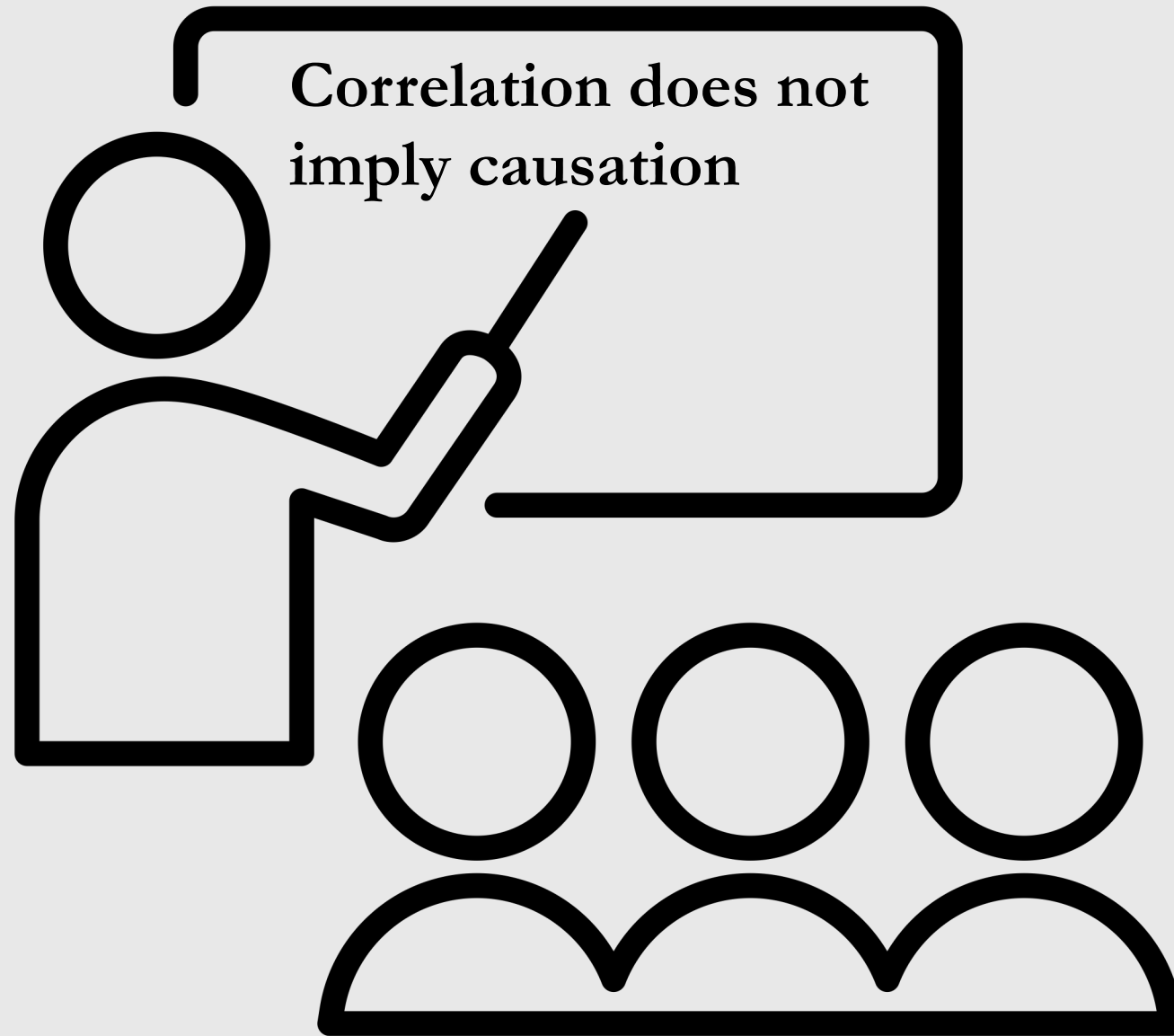
correlates with

**Films Nicolas Cage appeared in**



tylervigen.com

<https://www.tylervigen.com/spurious-correlations>



Then, what does imply causation?

Motivating example: Simpson's paradox

Correlation does not imply causation

**Then, what does imply causation?**

Causation in observational studies

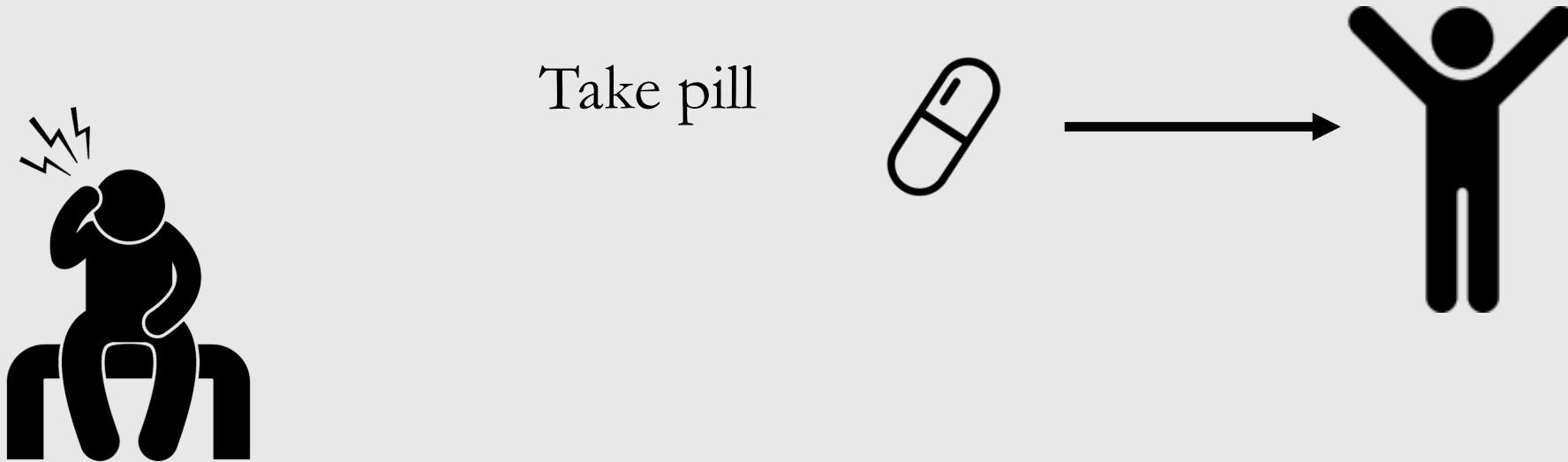
# Potential outcomes: intuition

Inferring the effect of treatment/policy on some outcome



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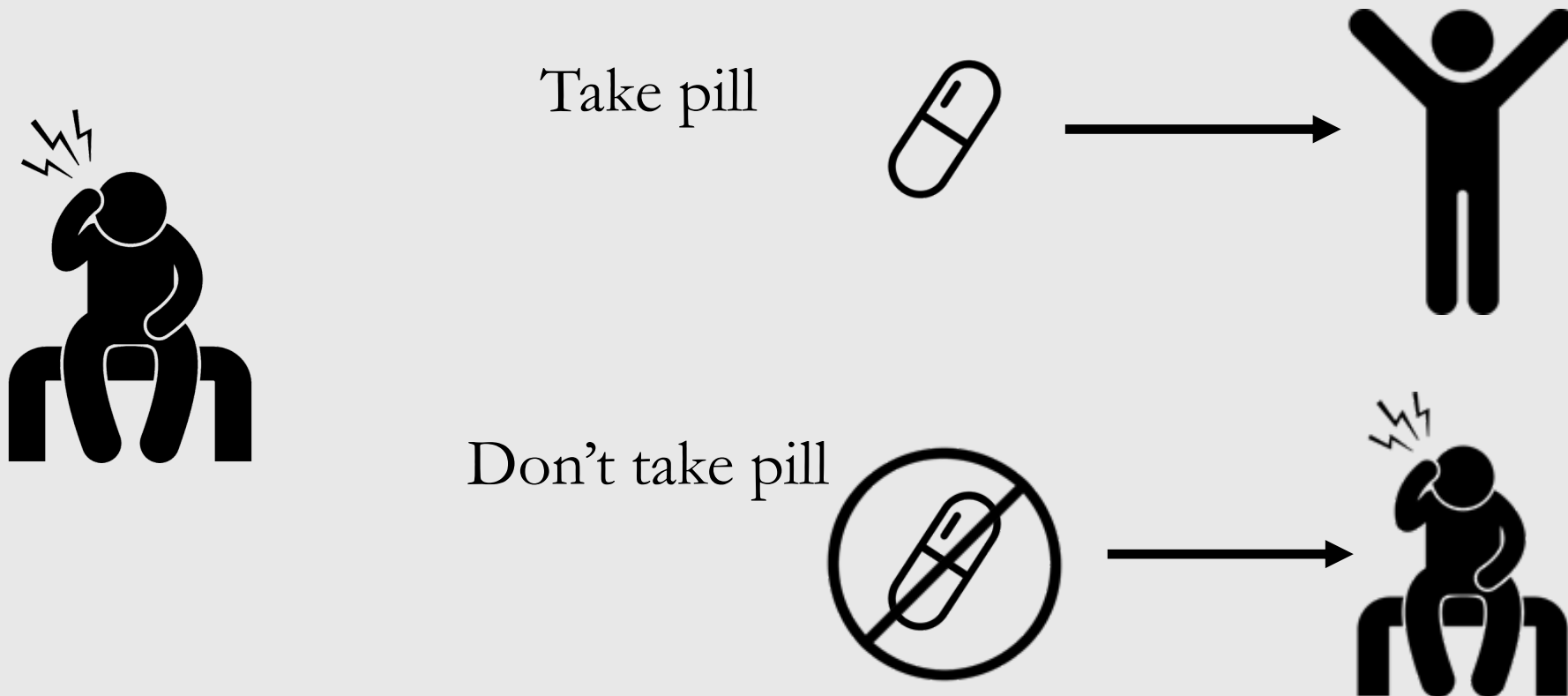
Inferring the effect of treatment/policy on some outcome





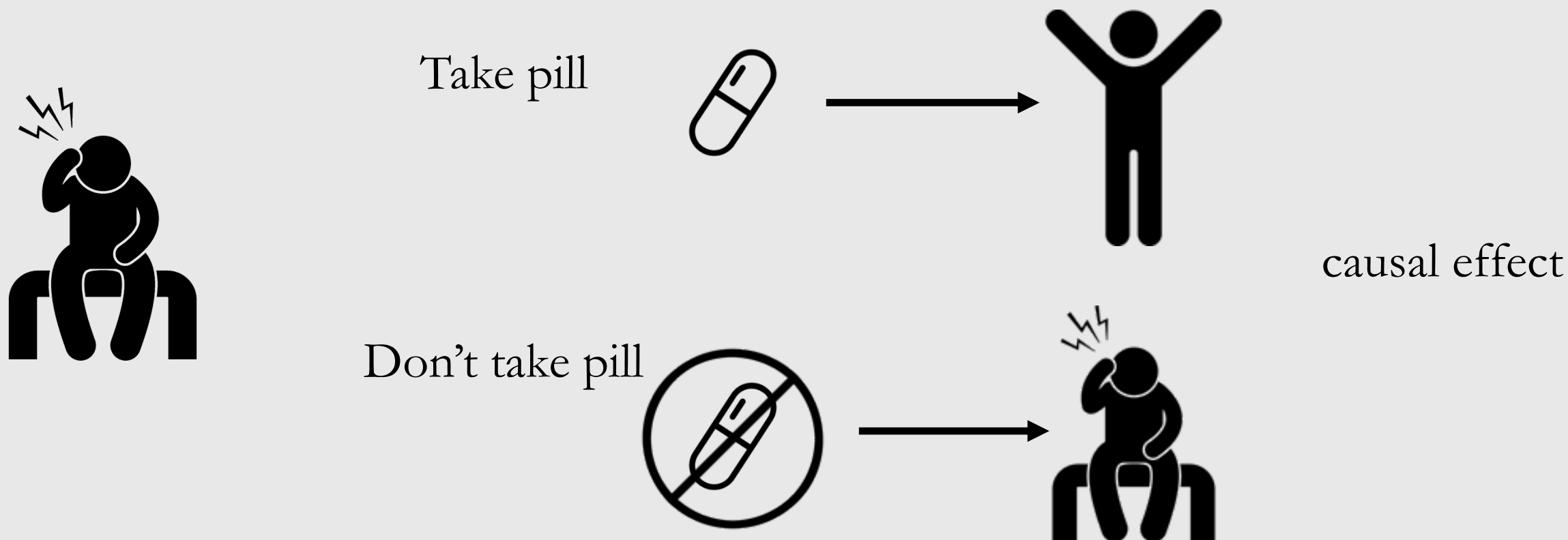
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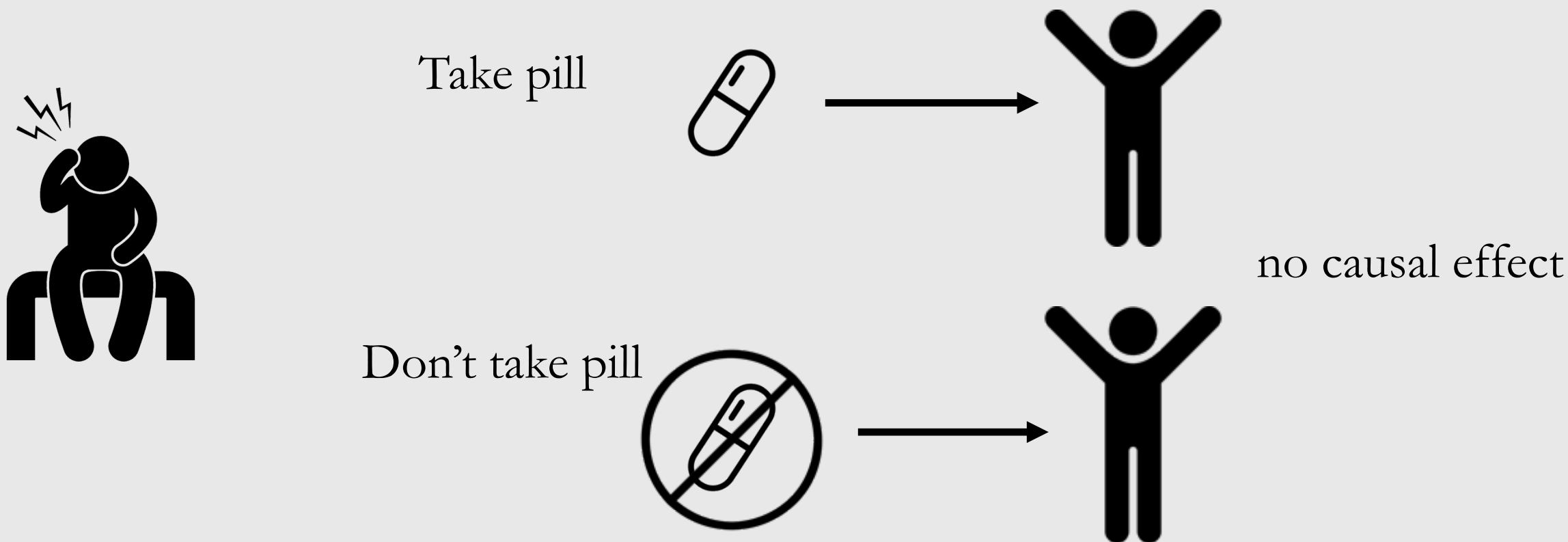
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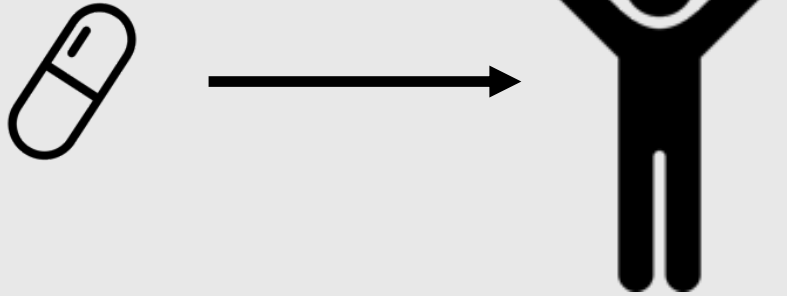
# Potential outcomes: intuition

Inferring the effect of treatment/policy on some outcome



# Potential outcomes: notation

$\text{do}(T = 1)$

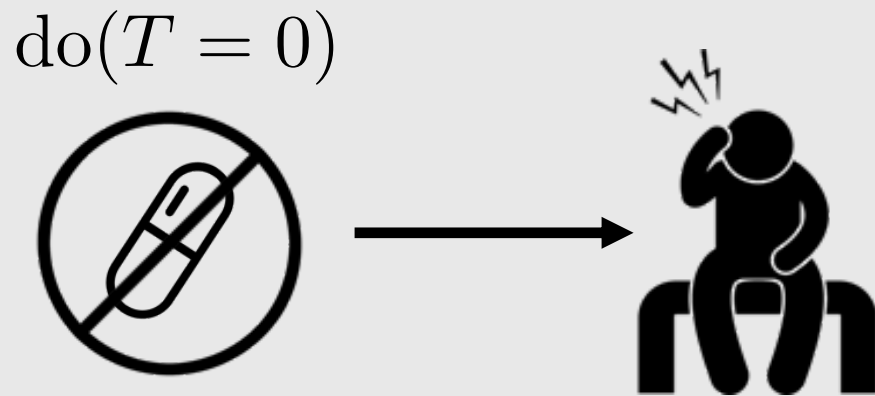
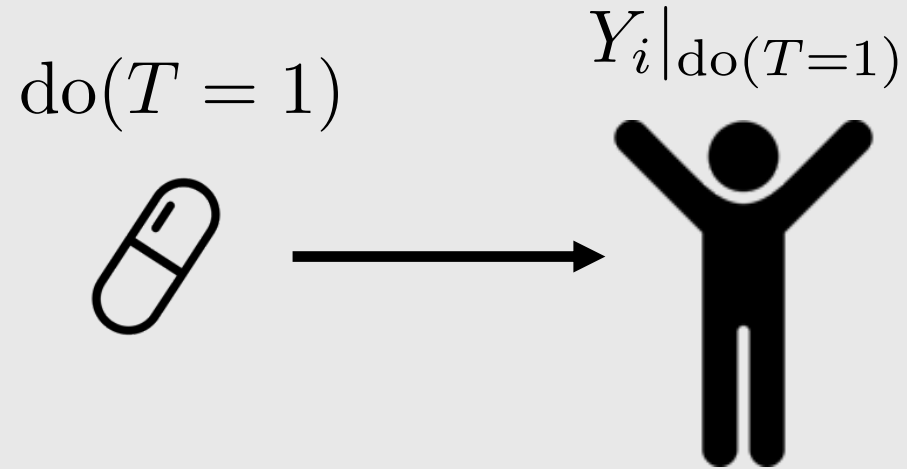


$\text{do}(T = 0)$



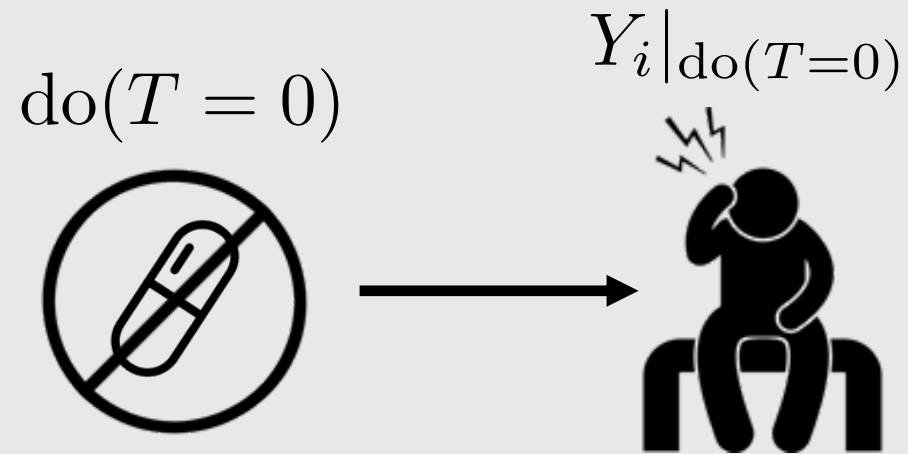
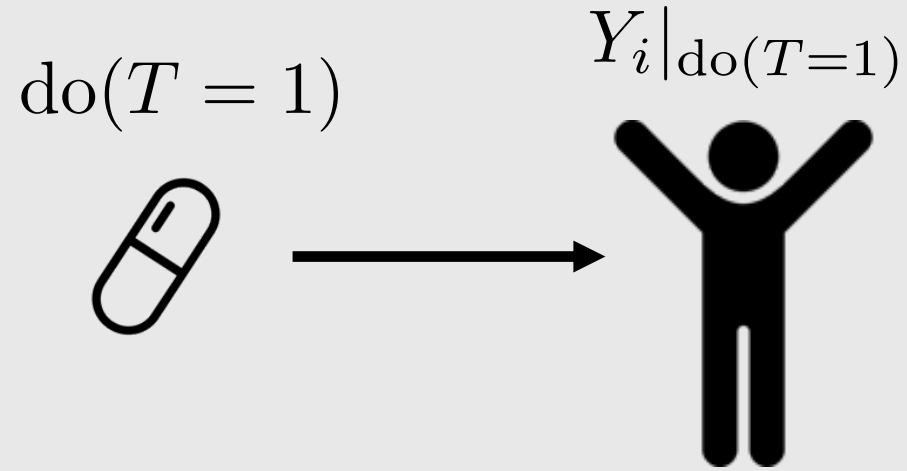
$T$  : observed treatment  
 $Y$  : observed outcome

# Potential outcomes: notation



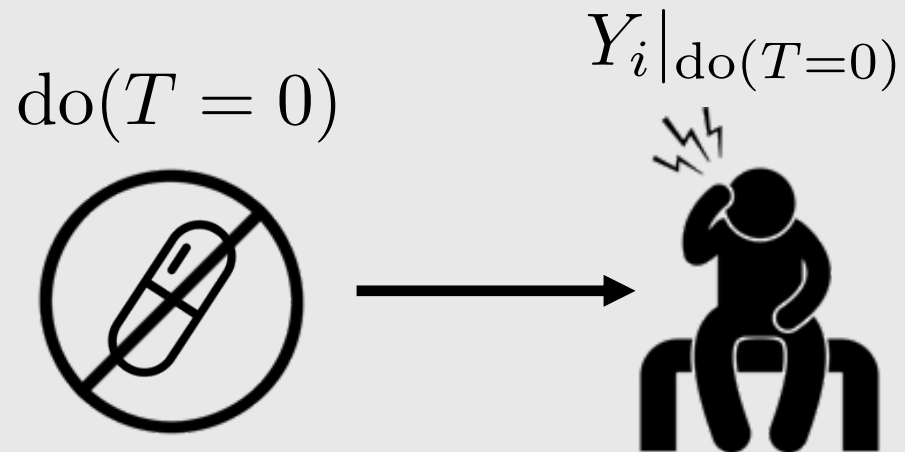
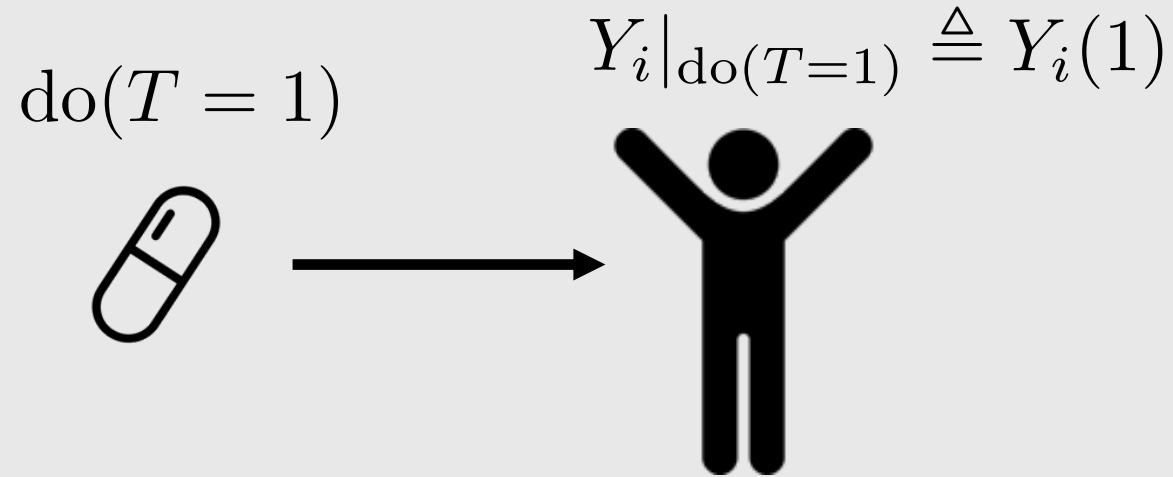
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# Potential outcomes: notation



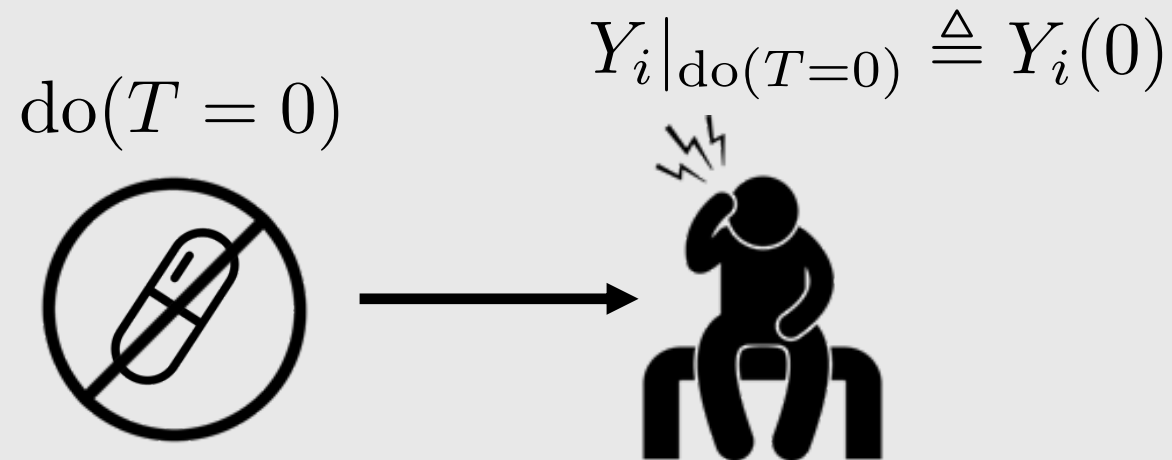
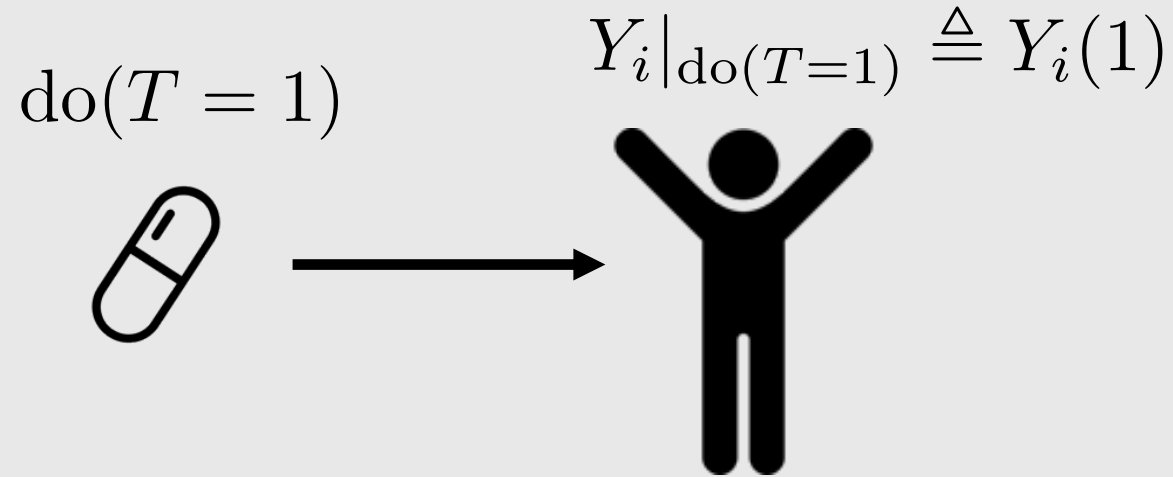
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# Potential outcomes: notation



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 $i$  : used in subscript to denote a specific unit/individual  
 $Y_i(1)$  : potential outcome under treatment

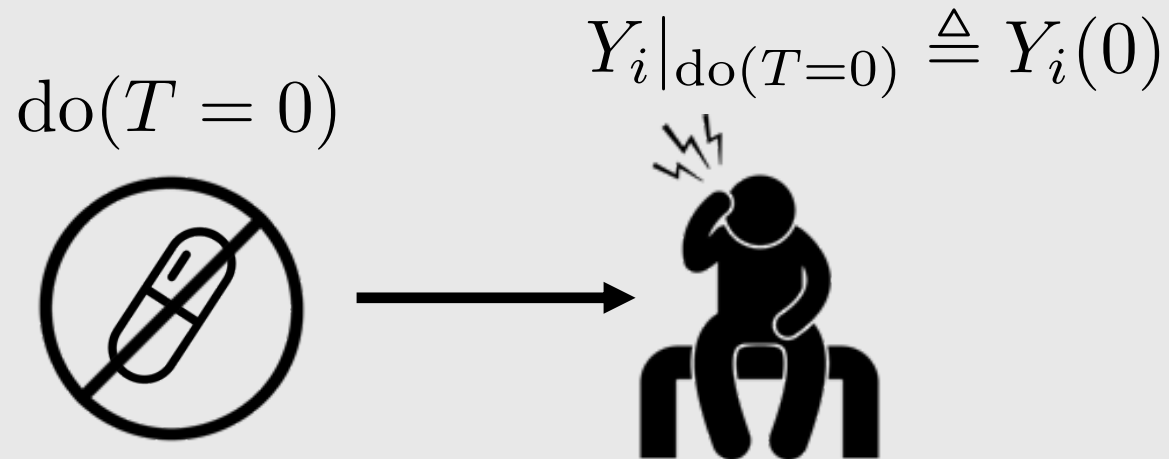
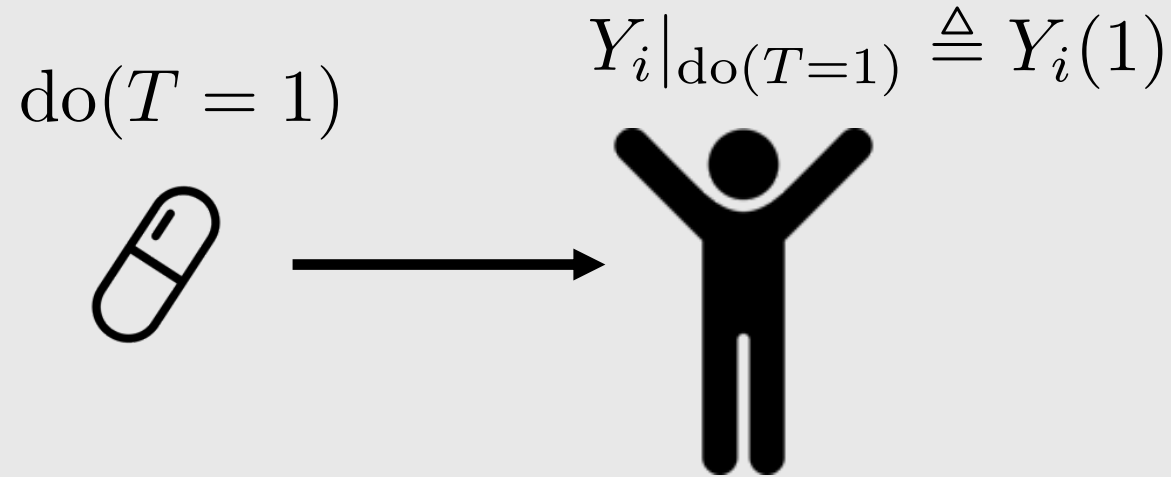
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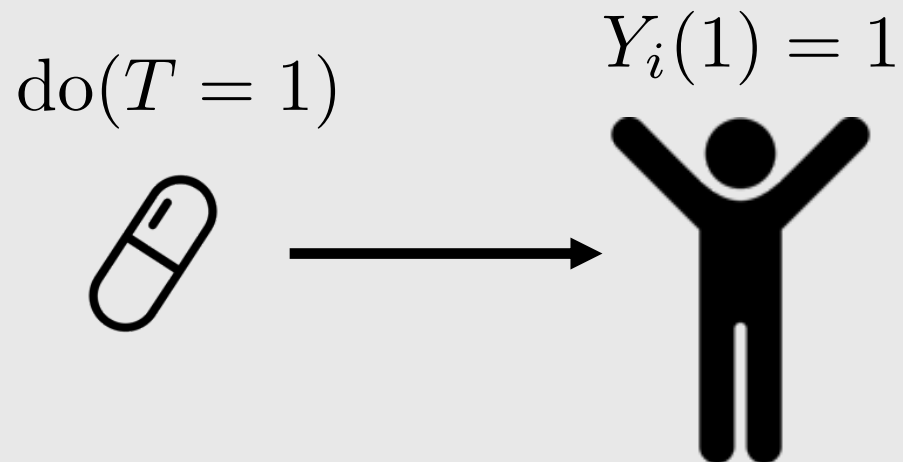
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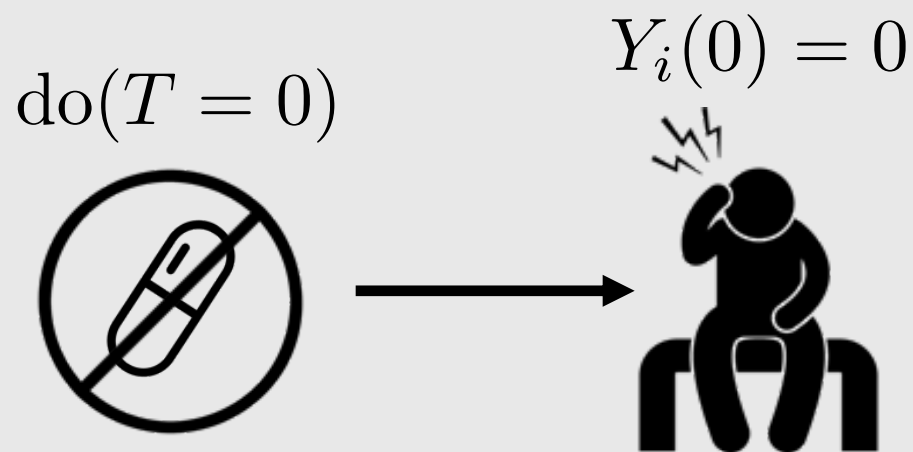
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**Causal effect**  
 $Y_i(1) - Y_i(0)$

# Fundamental problem of causal inference



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 $Y$  : observed outcome  
 $i$  : used in subscript to denote a specific unit/individual  
 $Y_i(1)$  : potential outcome under treatment  
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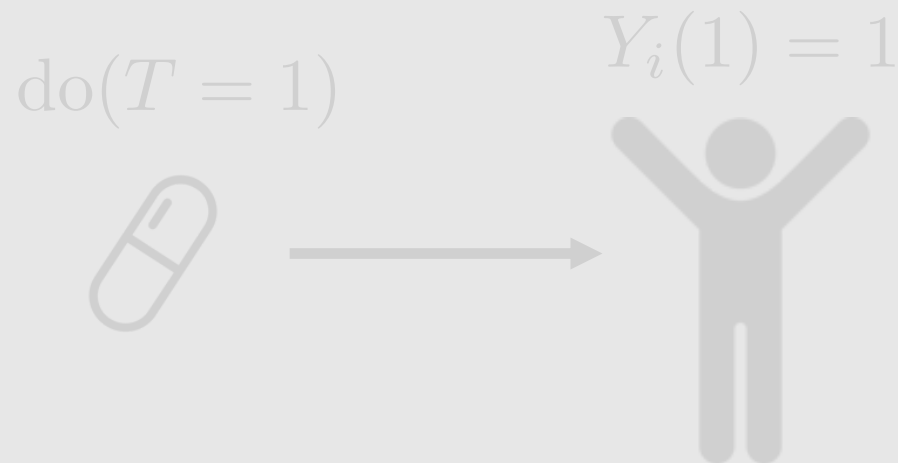


**Causal effect**

$$Y_i(1) - Y_i(0) = 1$$

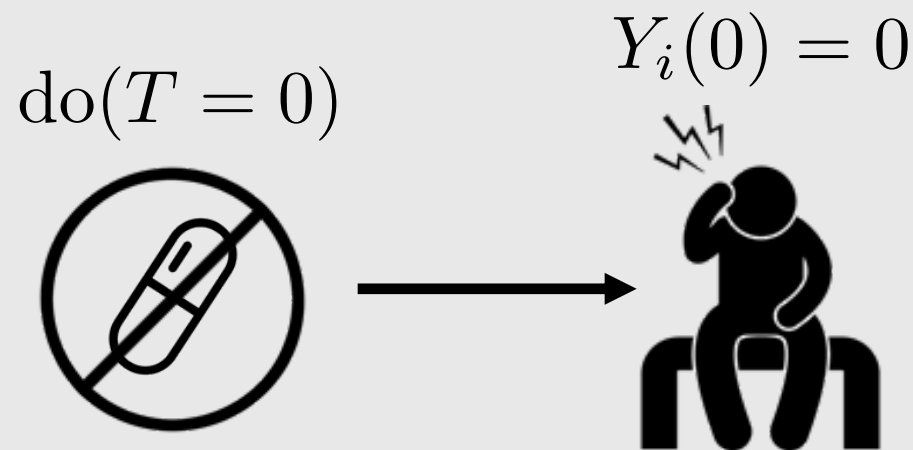
# Fundamental problem of causal inference

Counterfactual



- $T$  : observed treatment
- $Y$  : observed outcome
- $i$  : used in subscript to denote a specific unit/individual
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- $Y_i(0)$  : potential outcome under no treatment

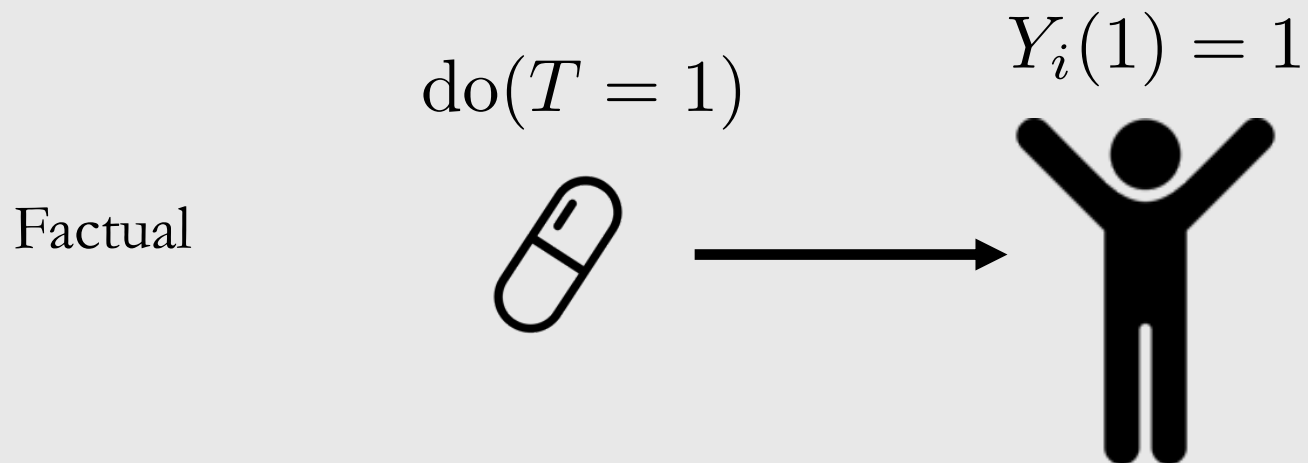
Factual



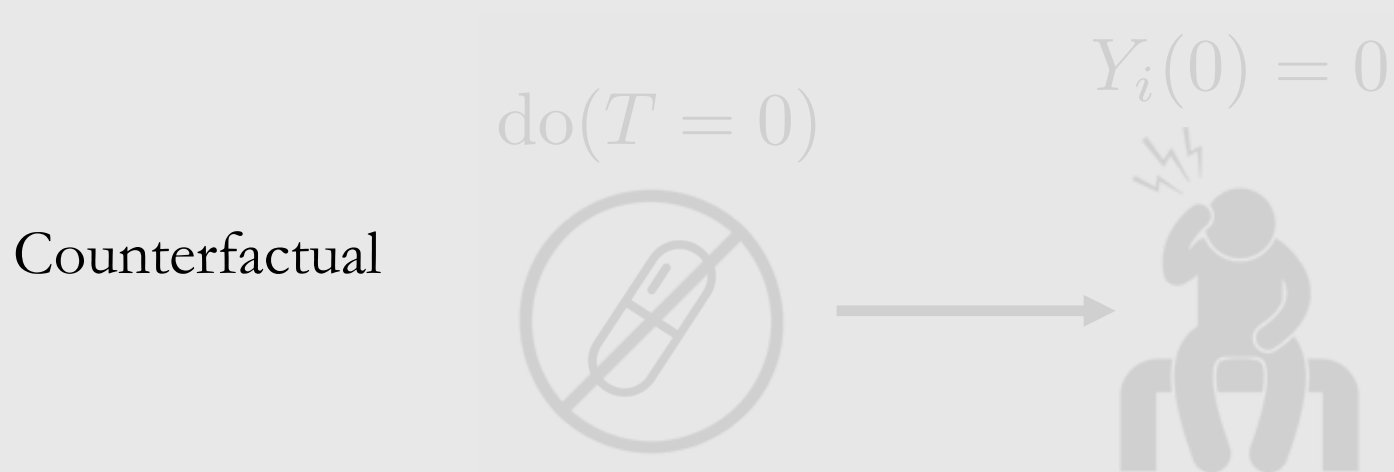
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# Fundamental problem of causal inference



$T$  : observed treatment  
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**Causal effect**

$$Y_i(1) - Y_i(0) = 1$$

# Average treatment effect (ATE)

Individual treatment effect (ITE):  $Y_i(1) - Y_i(0)$

$T$  : observed treatment

$Y$  : observed outcome

$i$  : used in subscript to denote a  
specific unit/individual

$Y_i(1)$  : potential outcome under treatment

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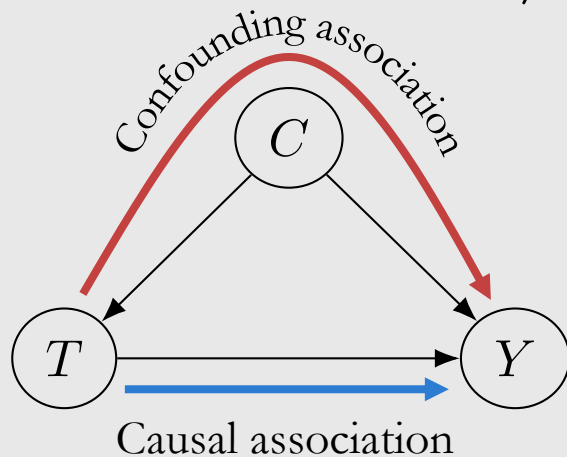
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Recall: correlation does not imply causation



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$T$  : observed treatment

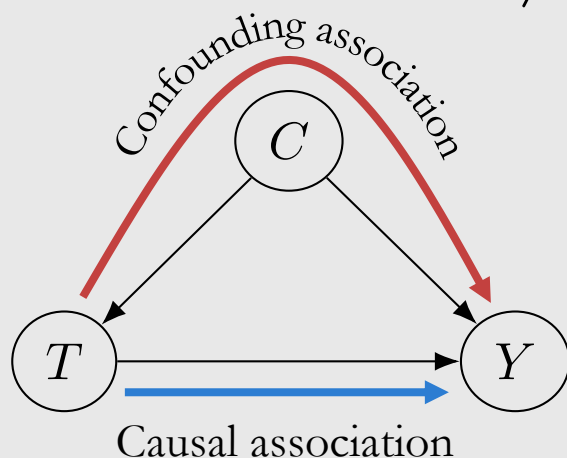
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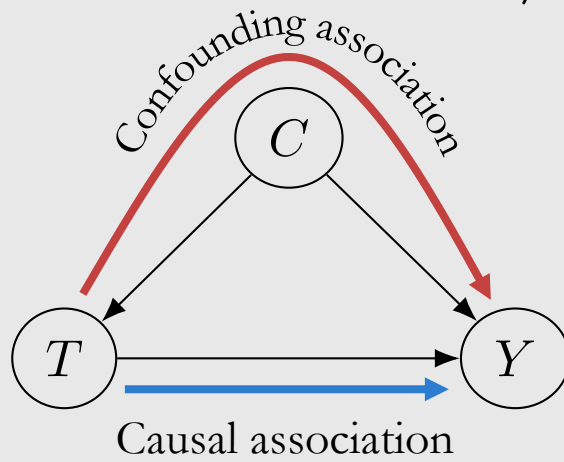
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Average treatment effect (ATE):

$$\mathbb{E}[Y_i(1) - Y_i(0)] = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$
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$T$  : observed treatment  
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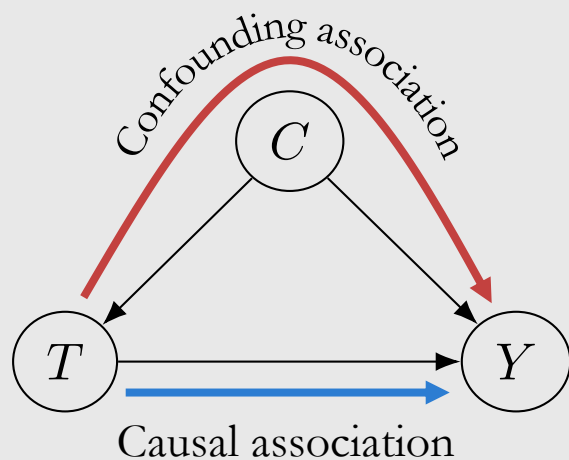


Recall: correlation does not imply causation

# Randomized control trials (RCTs)

ATE when there is confounding:

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \neq \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$$



$T$  : observed treatment

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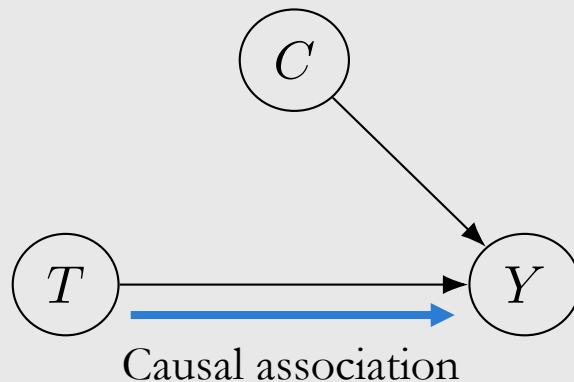
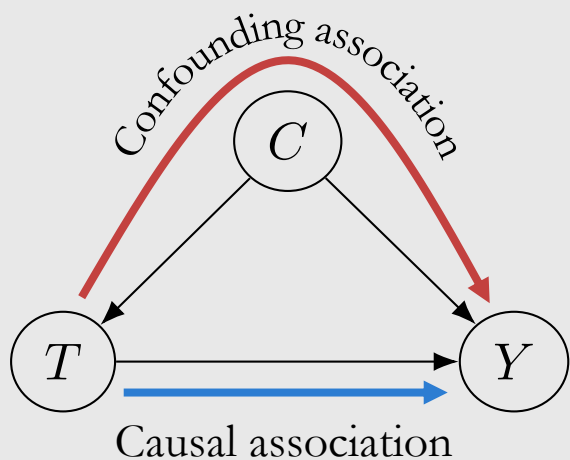
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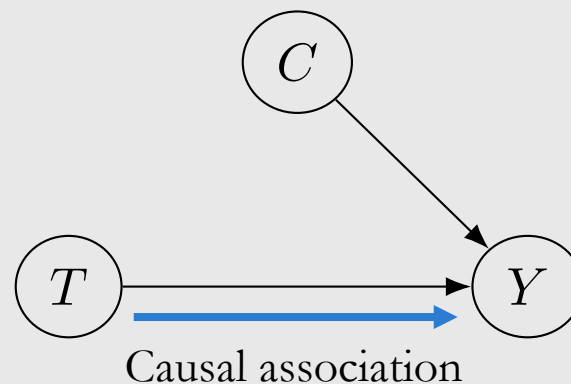
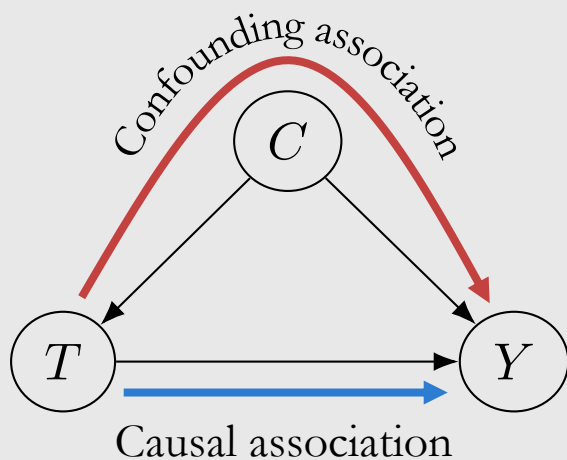
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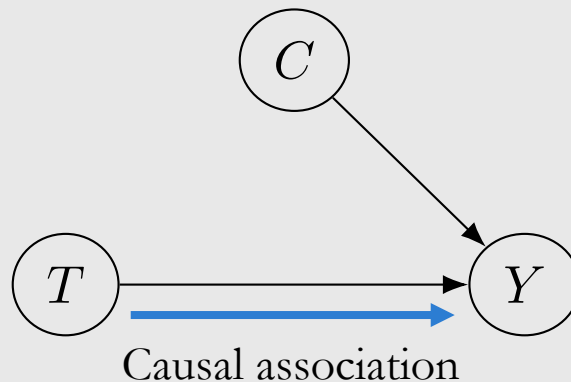
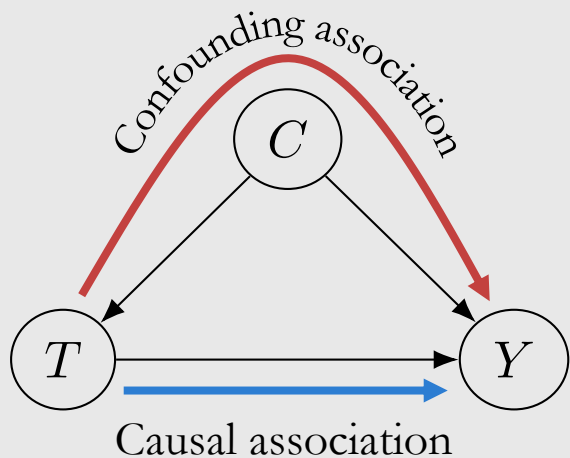
RCTs: experimenter randomizes subjects into treatment group or control group

1.  $T$  cannot have any causal parents

# Randomized control trials (RCTs)

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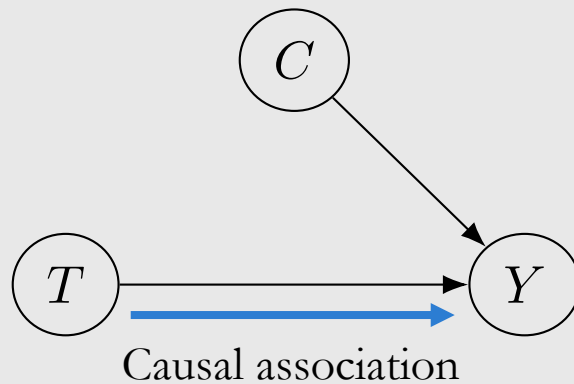
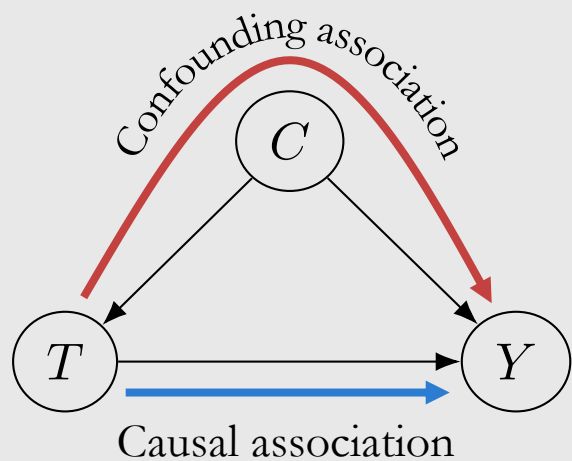
RCTs: experimenter randomizes subjects into treatment group or control group

1.  $T$  cannot have any causal parents
2. Groups are comparable

# Randomized control trials (RCTs)

ATE when there is confounding:

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \neq \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$$



$T$  : observed treatment

$Y$  : observed outcome

$i$  : used in subscript to denote a specific unit/individual

$Y_i(1)$  : potential outcome under treatment

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$Y(t)$  : population-level potential outcome

ATE when there is **no** confounding (e.g. RCTs):

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$$

RCTs: experimenter randomizes subjects into treatment group or control group

1. T cannot have any causal parents
2. Groups are comparable

Motivating example: Simpson's paradox

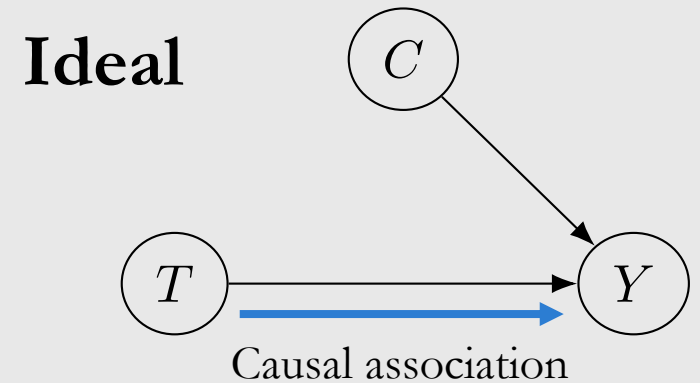
Correlation does not imply causation

Then, what does imply causation?

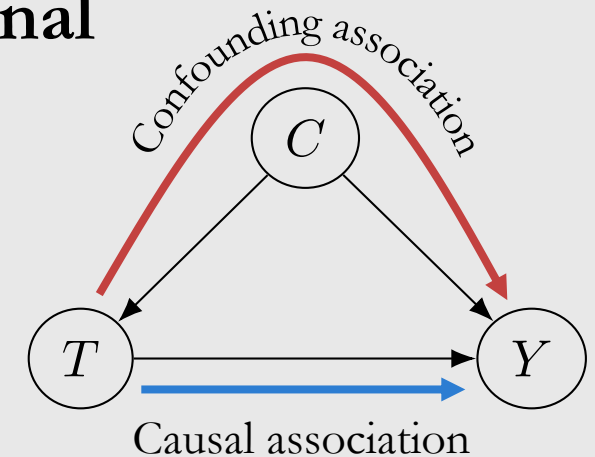
**Causation in observational studies**



# Observational studies



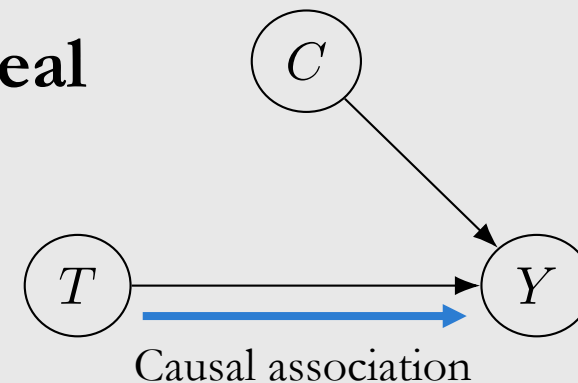
## Observational studies



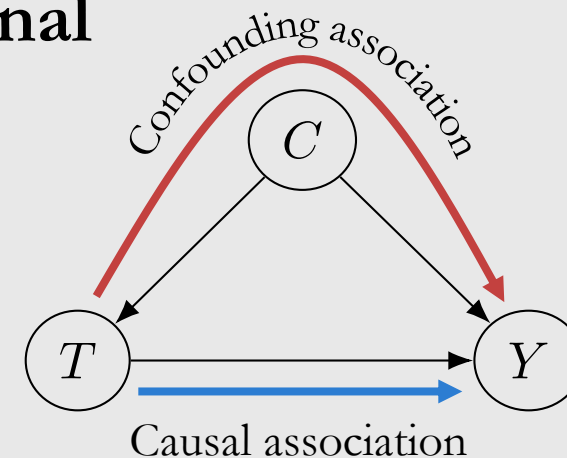
# Observational studies

Can't always randomize treatment

**Ideal**



**Observational studies**

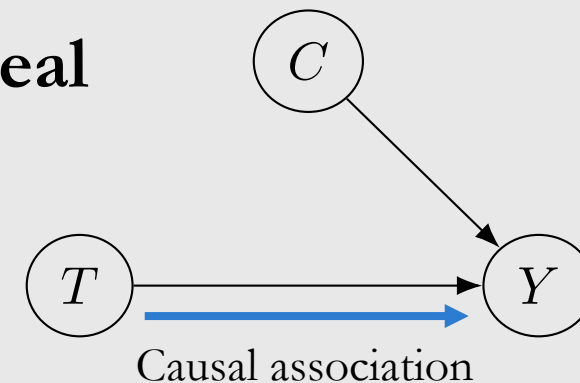


# Observational studies

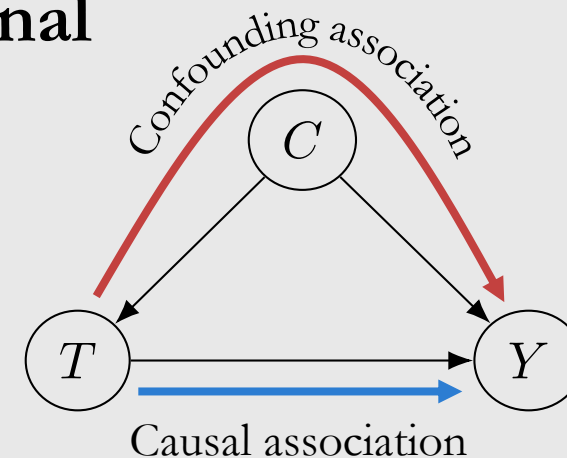
Can't always randomize treatment

- **Ethical reasons** (e.g. unethical to randomize people to smoke for measuring effect on lung cancer)

**Ideal**



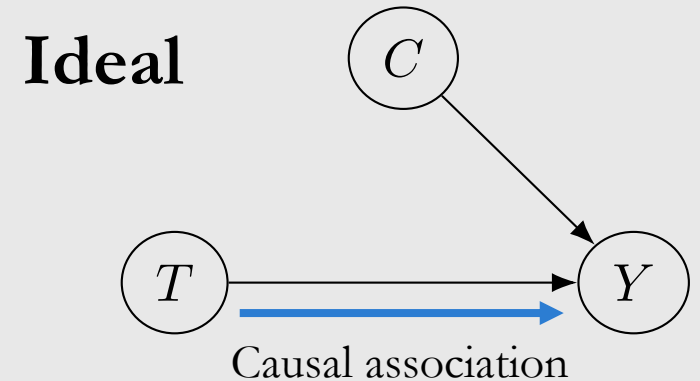
**Observational studies**



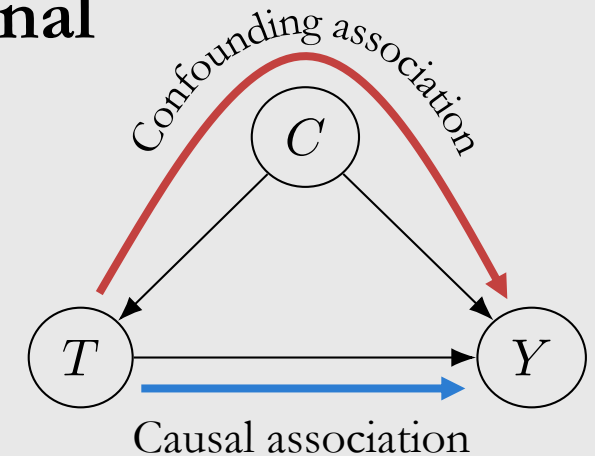
# Observational studies

Can't always randomize treatment

- **Ethical reasons** (e.g. unethical to randomize people to smoke for measuring effect on lung cancer)
- **Infeasibility** (e.g. can't randomize countries into communist/capitalist systems to measure effect on GDP)



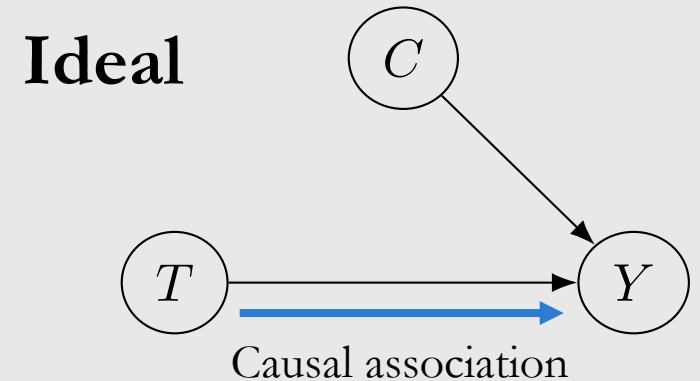
**Observational studies**



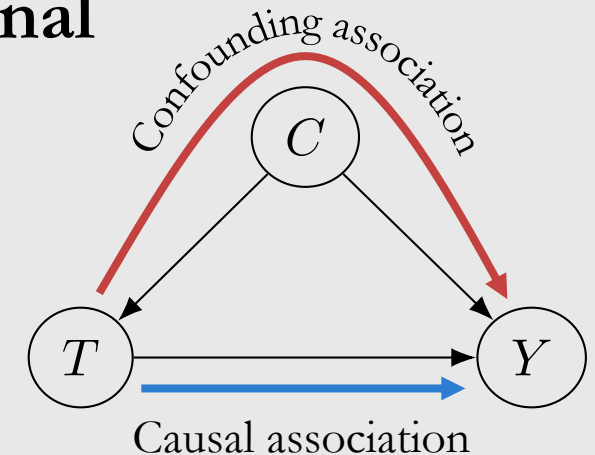
# Observational studies

Can't always randomize treatment

- **Ethical reasons** (e.g. unethical to randomize people to smoke for measuring effect on lung cancer)
- **Infeasibility** (e.g. can't randomize countries into communist/capitalist systems to measure effect on GDP)
- **Impossibility** (e.g. can't change a living person's DNA at birth for measuring effect on breast cancer)

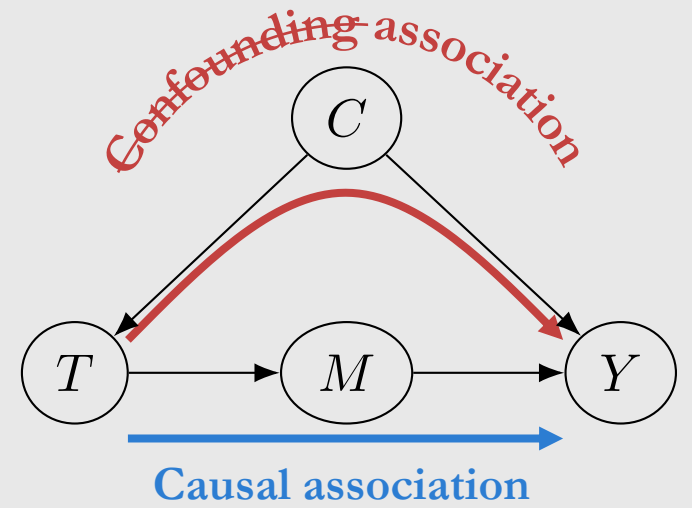


**Observational studies**



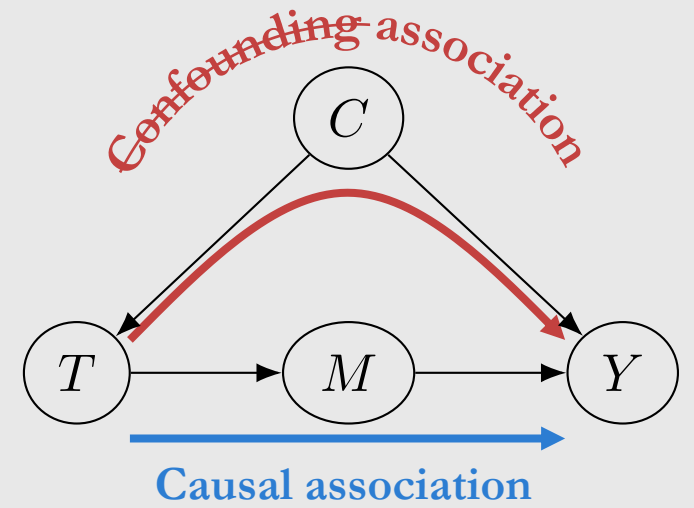
How do we measure causal effects in observational studies?

# Solution: adjust/control for confounders



# Solution: adjust/control for confounders

Adjust/control for the right variables  $W$ .



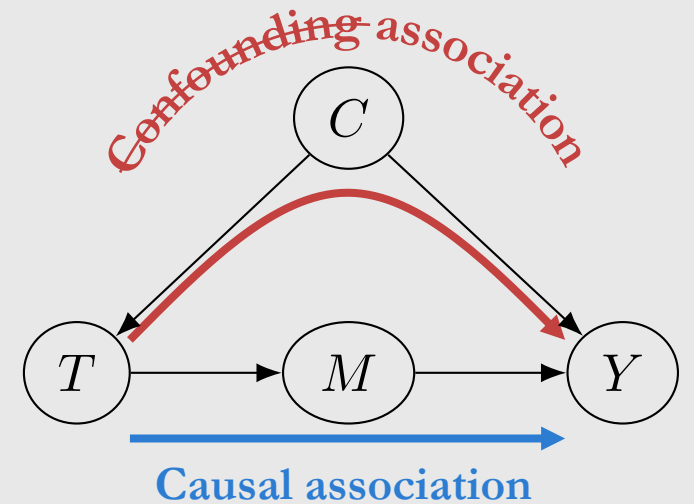


# Solution: adjust/control for confounders

Adjust/control for the right variables  $W$ .

If  $W$  is a sufficient adjustment set, we have

$$\mathbb{E}[Y(t)|W = w] \triangleq \mathbb{E}[Y|\text{do}(T = t), W = w] = \mathbb{E}[Y|t, w]$$

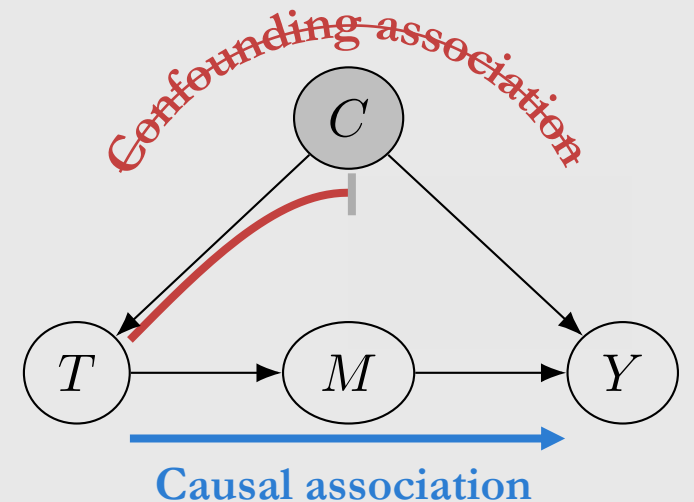


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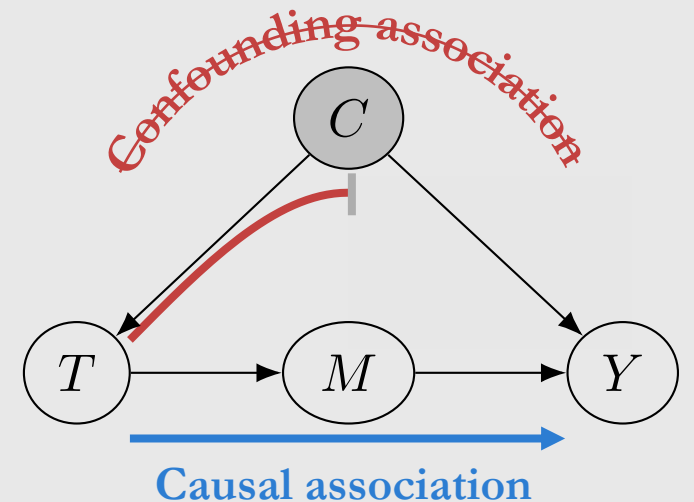
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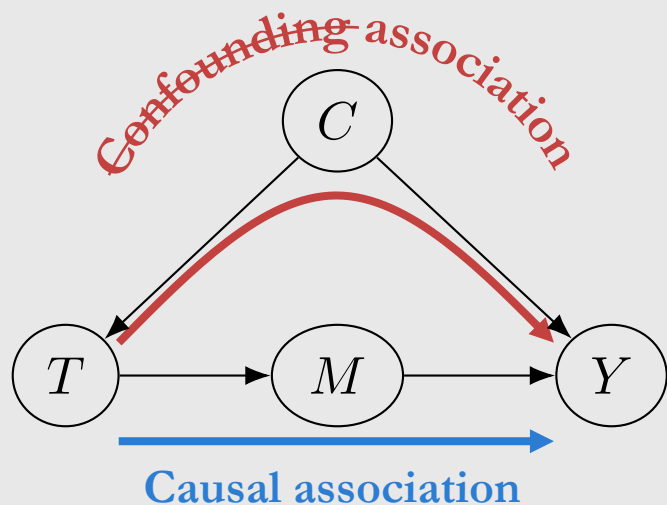
$$\mathbb{E}[Y(t) | \underline{W} = w] \triangleq \mathbb{E}[Y | \text{do}(T = t), \underline{W} = w] = \mathbb{E}[Y | t, \underline{w}]$$

$$\mathbb{E}[Y(t)] \triangleq \mathbb{E}[Y | \text{do}(T = t)] = \mathbb{E}_W \mathbb{E}[Y | t, W]$$



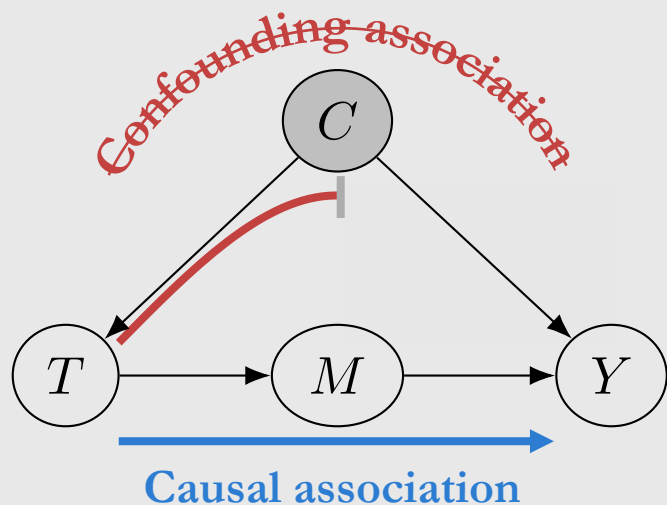
# Solution: backdoor adjustment

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_W \mathbb{E}[Y|t, W]$$



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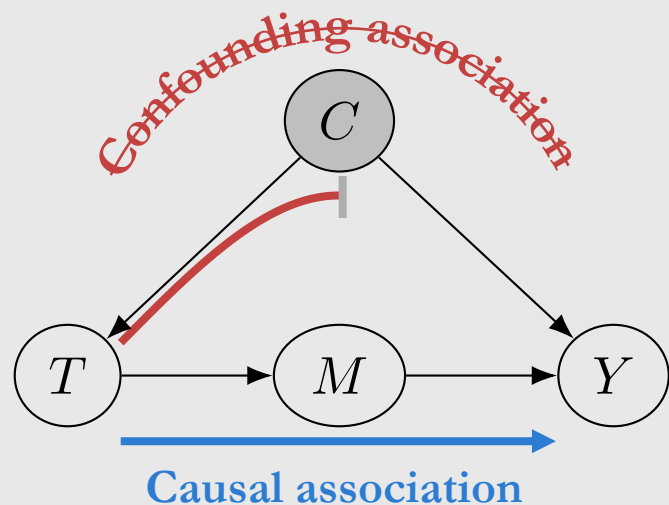
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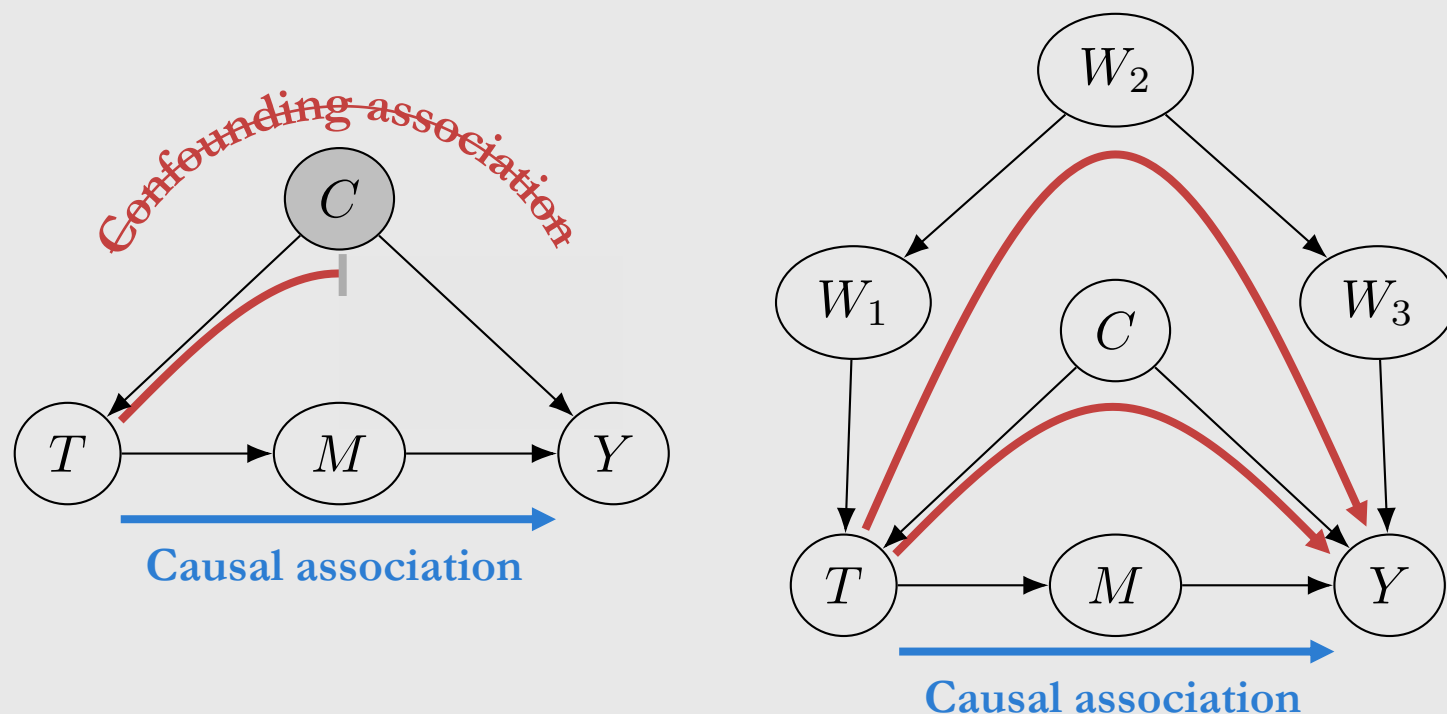
Shaded nodes are examples of sufficient adjustment sets  $W$



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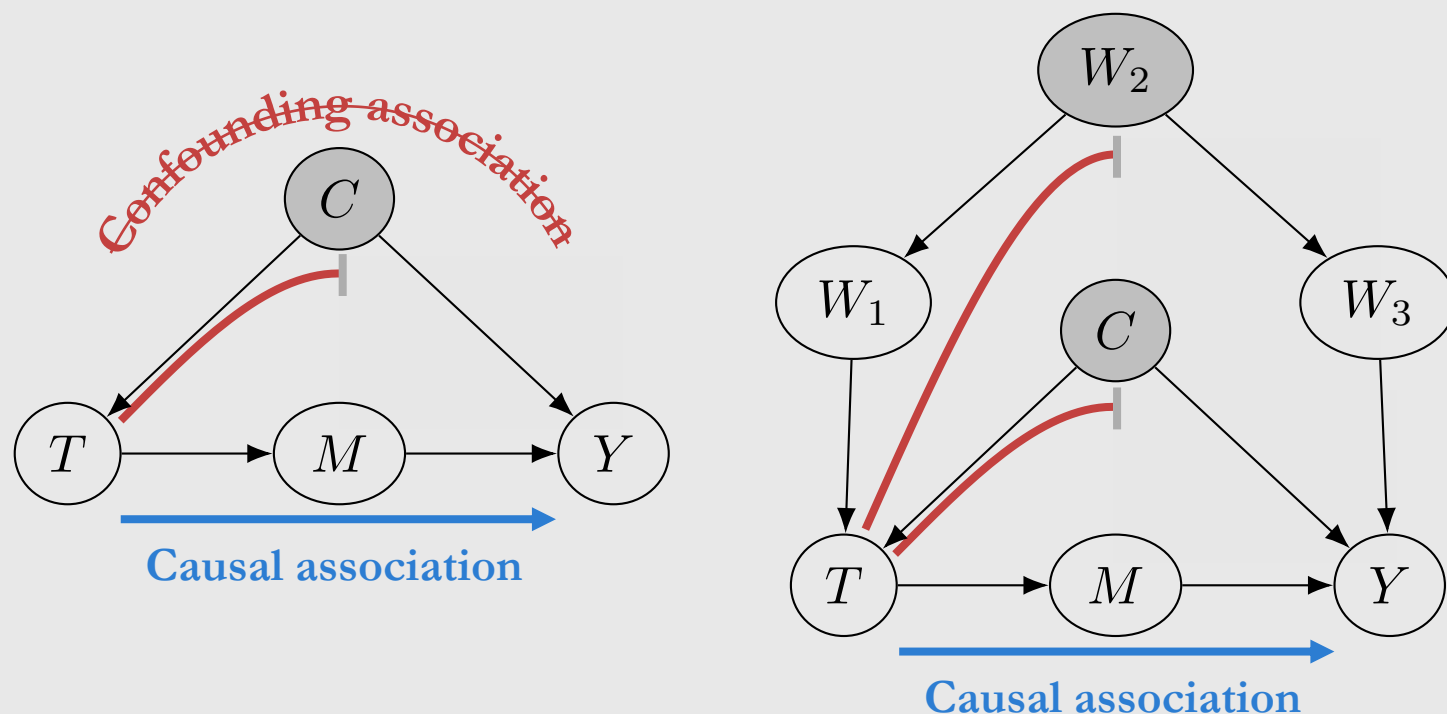
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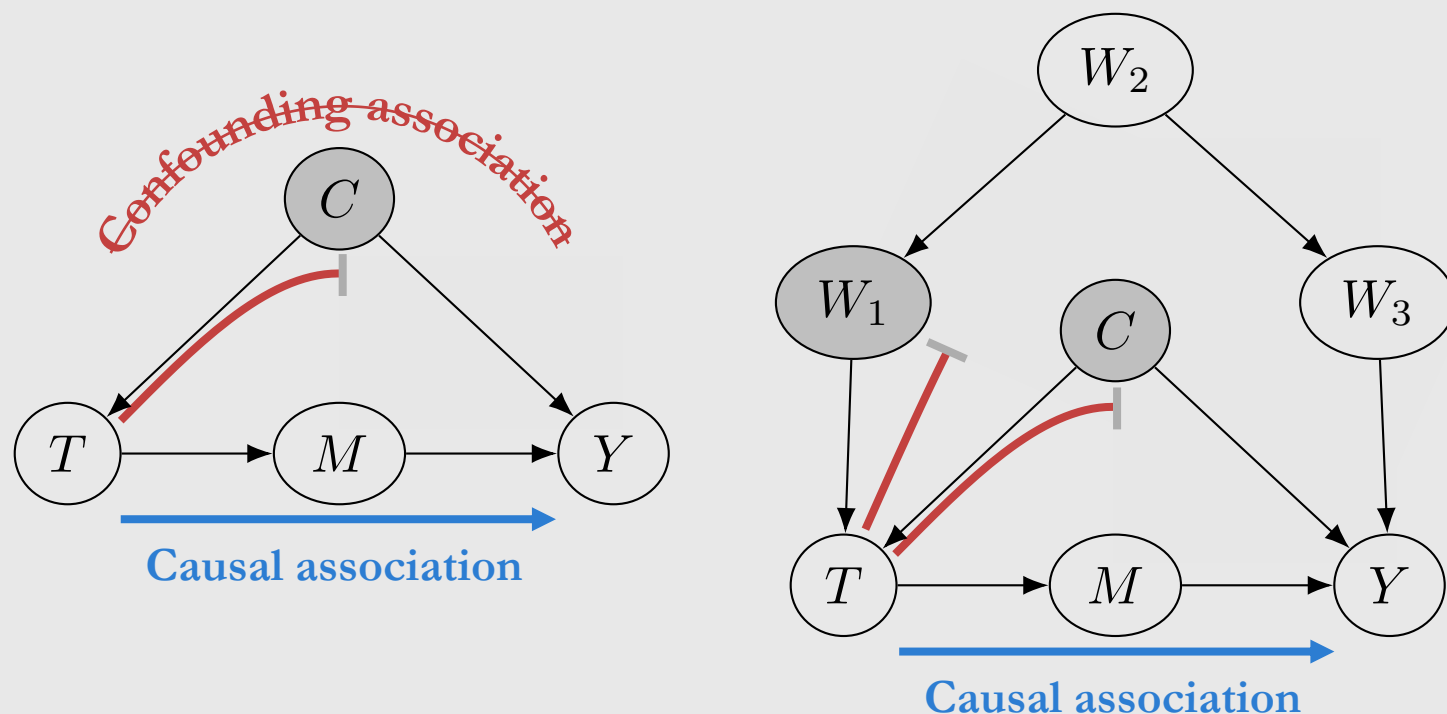




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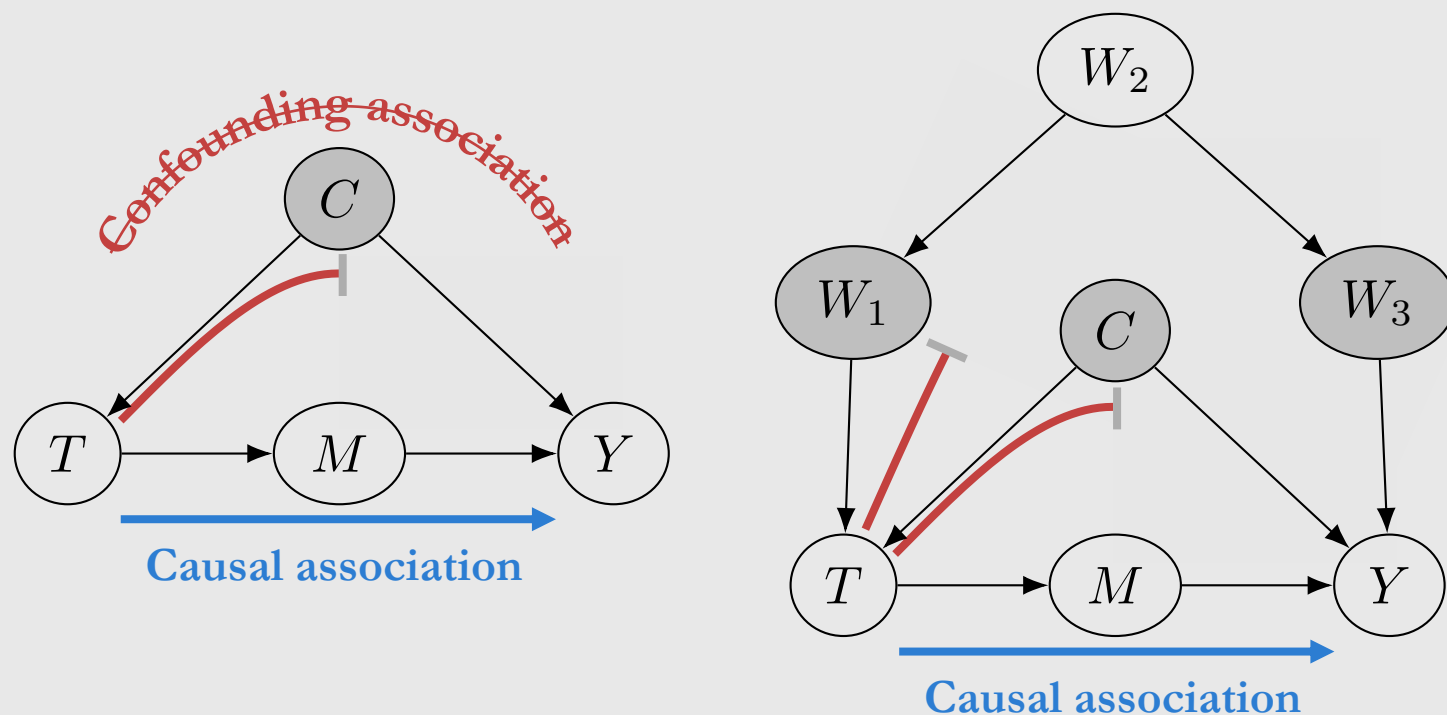
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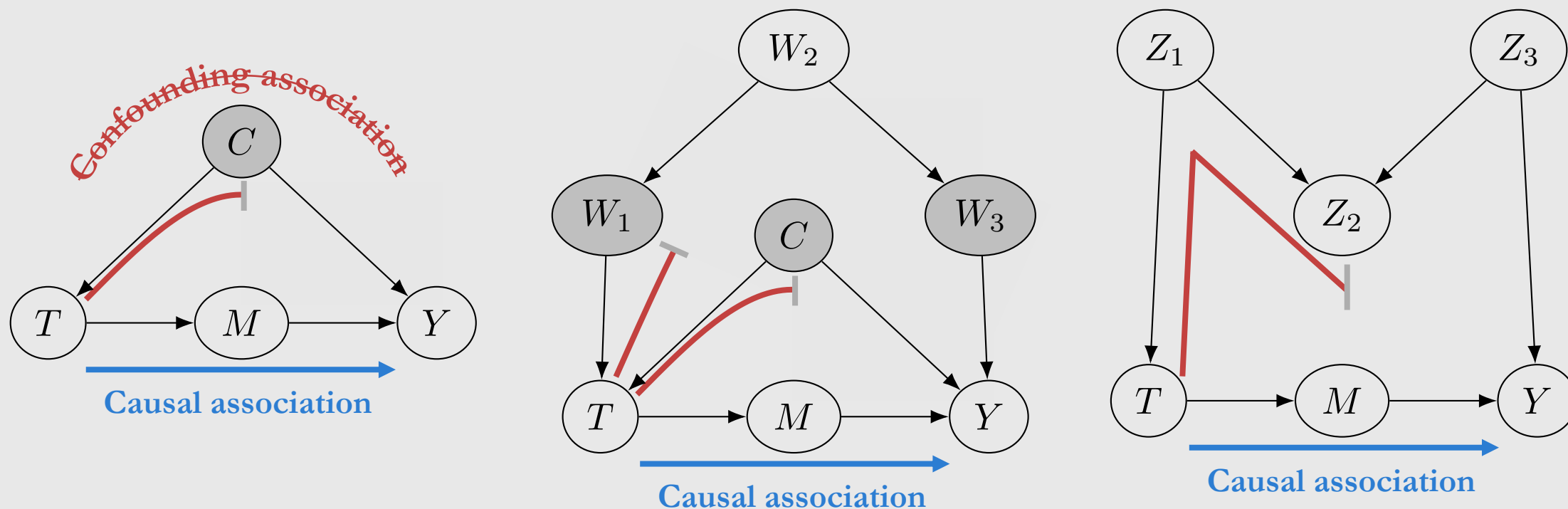
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Shaded nodes are examples of sufficient adjustment sets  $W$



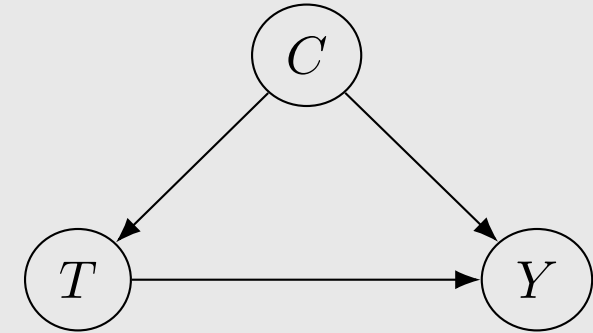
# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C]$$

## Condition

		Condition		
		Mild	Severe	Total
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)
	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)
		$\mathbb{E}[Y t, C = 0]$	$\mathbb{E}[Y t, C = 1]$	$\mathbb{E}[Y t]$

## Causal Graph

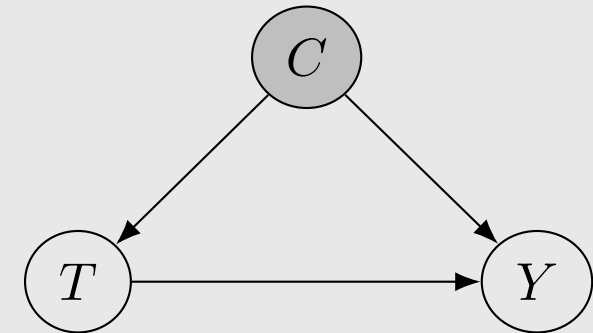


# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C]$$

		Condition		
		Mild	Severe	Total
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)
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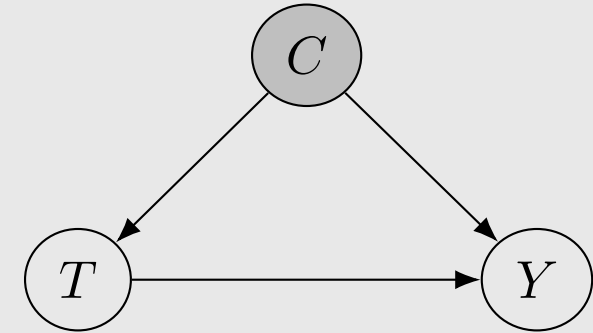
Causal Graph



# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



Condition

Treatment

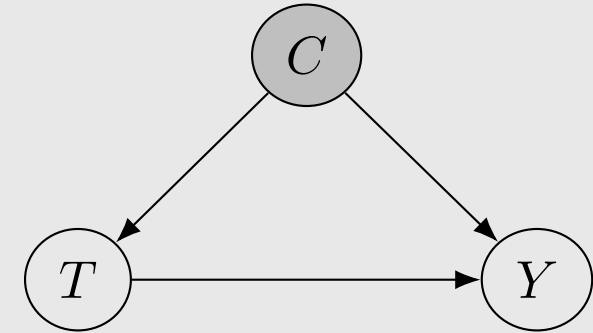
	Mild	Severe	Total
A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)
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$\mathbb{E}[Y|t, C = 0]$     $\mathbb{E}[Y|t, C = 1]$     $\mathbb{E}[Y|t]$

# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



Condition

Treatment

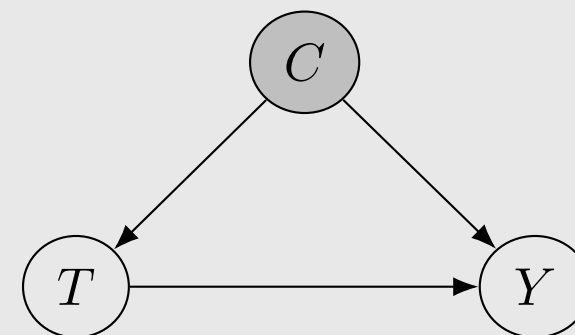
	Mild	Severe	Total	Causal
A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)	19.4%
B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)	<b>12.9%</b>

$\mathbb{E}[Y|t, C = 0]$    
  $\mathbb{E}[Y|t, C = 1]$    
  $\mathbb{E}[Y|t]$    
  $\mathbb{E}[Y|\text{do}(t)]$

# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



		Condition			Causal
		Mild	Severe	Total	
<b>Treatment</b>	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)	19.4%
	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)	<b>12.9%</b>
		$\mathbb{E}[Y t, C = 0]$	$\mathbb{E}[Y t, C = 1]$	$\mathbb{E}[Y t]$	$\mathbb{E}[Y \text{do}(t)]$

$$\frac{1450}{2050} (0.15) + \frac{600}{2050} (0.30) \approx 0.194$$

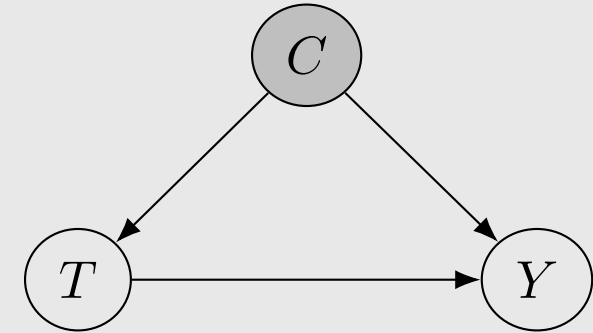
$$\frac{1450}{2050} (0.10) + \frac{600}{2050} (0.20) \approx 0.129$$



# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



		Condition		Total	Causal
		Mild	Severe		
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)	19.4%
	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)	<b>12.9%</b>
		$\mathbb{E}[Y t, C = 0]$	$\mathbb{E}[Y t, C = 1]$	$\mathbb{E}[Y t]$	$\mathbb{E}[Y \text{do}(t)]$

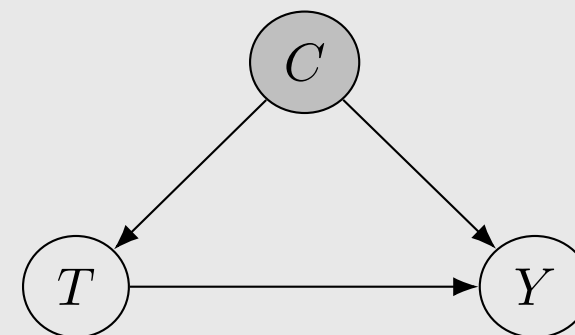
$$\frac{1450}{2050} (0.15) + \frac{600}{2050} (0.30) \approx 0.194$$

$$\frac{1450}{2050} (0.10) + \frac{600}{2050} (0.20) \approx 0.129$$

# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



		Condition			Causal
		Mild	Severe	Total	
Treatment	A	15% (210/ <u>1400</u> )	30% (30/100)	<b>16%</b> (240/1500)	19.4%
	B	<b>10%</b> (5/ <u>50</u> )	<b>20%</b> (100/500)	19% (105/550)	<b>12.9%</b>
		$\mathbb{E}[Y t, C = 0]$	$\mathbb{E}[Y t, C = 1]$	$\mathbb{E}[Y t]$	$\mathbb{E}[Y \text{do}(t)]$

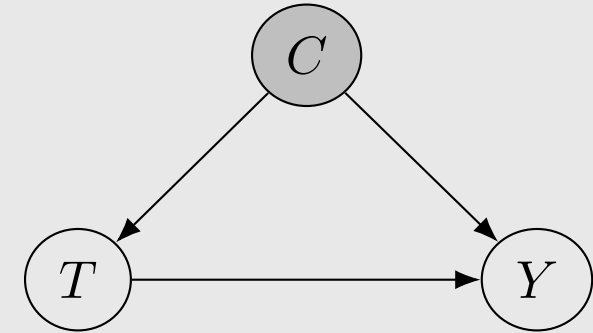
$$\frac{1450}{2050} (0.15) + \frac{600}{2050} (0.30) \approx 0.194$$

$$\frac{1450}{2050} (0.10) + \frac{600}{2050} (0.20) \approx 0.129$$

# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



		Condition			Causal
		Mild	Severe	Total	
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)	19.4%
	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)	<b>12.9%</b>
		$\mathbb{E}[Y t, C = 0]$	$\mathbb{E}[Y t, C = 1]$	$\mathbb{E}[Y t]$	$\mathbb{E}[Y \text{do}(t)]$

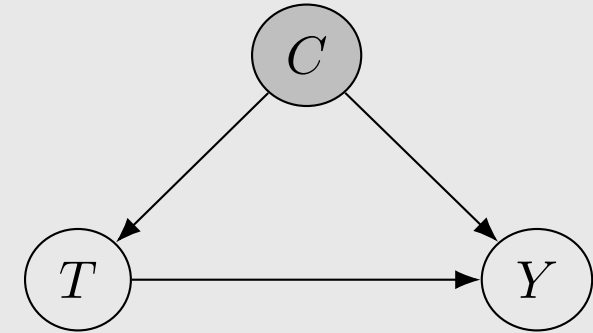
$$\frac{1450}{2050} (0.15) + \frac{600}{2050} (0.30) \approx 0.194$$

$$\frac{1450}{2050} (0.10) + \frac{600}{2050} (0.20) \approx 0.129$$

# Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T = t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



		Condition			Causal
		Mild	Severe	Total	
<b>Treatment</b>	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)	19.4%
	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)	<b>12.9%</b>
		$\mathbb{E}[Y t, C = 0]$	$\mathbb{E}[Y t, C = 1]$	$\mathbb{E}[Y t]$	$\mathbb{E}[Y \text{do}(t)]$

$$\frac{1450}{2050} (0.15) + \frac{600}{2050} (0.30) \approx 0.194$$

$$\frac{1450}{2050} (0.10) + \frac{600}{2050} (0.20) \approx 0.129$$

# Application to the COVID-27 example

		Condition			Causal	Naive
		Mild	Severe	Total		
Treatment	A	15% (210/1400)	30% (30/100)	<b>16%</b> (240/1500)	19.4%	$\frac{1400}{1500} (0.15) + \frac{100}{1500} (0.30) = 0.16$
	B	<b>10%</b> (5/50)	<b>20%</b> (100/500)	19% (105/550)	<b>12.9%</b>	$\frac{50}{550} (0.10) + \frac{500}{550} (0.20) = 0.19$
		$\mathbb{E}[Y t, C = 0]$	$\mathbb{E}[Y t, C = 1]$	$\mathbb{E}[Y t]$	$\mathbb{E}[Y do(t)]$	



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